During the last two decades, the role of monetary aggregates in monetary policy research has been strongly reduced. This paper aims to reassess and analyze the dynamic interactions between money, prices and economic activity in the case of the Republic of Macedonia. The first part of the paper simulates the property of the superneutrality of money, based on Sidrauski’s (1967) framework. The second part presents the money demand estimations on the monetary aggregate M2 for the period from 2002 to 2012, using the cointegration approach. Following Cziráky and Gillman (2006), we examine the validity of the Fisher equation in the case of Macedonia. The Fisher equation does not hold in the case of Macedonia, so the inflation rate must be included in the money demand specification. The estimated cointegration equation is in line with economic theory. The cointegration equation shows income elasticity less than unity (0.81), small and negative interest rate semi-elasticity (-0.17) and negative elasticity with respect to inflation. The short-run dynamics reveal that only 2.70% of the disequilibrium is corrected in a single quarter. The properties of stability imply that the M2 aggregate may serve as a proper policy indicator.

**Abstract**

This paper aims to reassess and analyze the dynamic interactions between money, prices and economic activity in the case of the Republic of Macedonia. The first part of the paper simulates the property of the superneutrality of money, based on Sidrauski’s (1967) framework. The second part presents the money demand estimations on the monetary aggregate M2 for the period from 2002 to 2012, using the cointegration approach. Following Cziráky and Gillman (2006), we examine the validity of the Fisher equation in the case of Macedonia. The Fisher equation does not hold in the case of Macedonia, so the inflation rate must be included in the money demand specification. The estimated cointegration equation is in line with economic theory. The cointegration equation shows income elasticity less than unity (0.81), small and negative interest rate semi-elasticity (-0.17) and negative elasticity with respect to inflation. The short-run dynamics reveal that only 2.70% of the disequilibrium is corrected in a single quarter. The properties of stability imply that the M2 aggregate may serve as a proper policy indicator.

**Keywords:** money-in-the-utility function, demand for money, cointegration

**JEL classification code:** E41, E17

**INTRODUCTION**

During the last two decades, the role of monetary aggregates in monetary policy research has been strongly reduced. This paper aims to reassess and analyze the dynamic interactions between money, prices and economic activity in the case of the Republic of Macedonia. The stability of money demand has significant implications on the actual conduct of monetary policy. If the relationship between the money and prices is stable, monetary aggregate targeting can be appropriate monetary strategy. At the same time, the unstable demand for money reduces the ability of monetary authorities to control inflation, which is the main goal of monetary policy in the Republic of Macedonia. The monetary authorities in the Republic of Macedonia targeted the growth rate of the monetary aggregate M1 during the period from 1992 to 1995. During this period, the relationship between the growth rate of the money supply and aggregate demand was very strong (Fetai 2008). However, the results regarding the main goal of the monetary policy were unsatisfactory: the inflation rate was still

---

* Mile Ivanov, MSc  
Faculty of Economics, Ss Cyril and Methodius University in Skopje  
ivanov.mile89@gmail.com

Mihail Petkovski, PhD  
Faculty of Economics, Ss Cyril and Methodius University in Skopje  
mihail.petkovski@eccf.ukim.edu.mk

Elena Naumovska, PhD  
Faculty of Economics, Ss Cyril and Methodius University in Skopje  
elenan@eccf.ukim.edu.mk
relatively high. During the period of monetary targeting, the demand for money was very unstable mainly due to high dollarization (Fetai, 2008). The Republic of Macedonia abandoned the monetary targeting regime in 1995 and implemented an exchange rate targeting regime. Under this regime, money became an endogenous variable, subordinated to the stability of the exchange rate. Following the process of financial development and deepening monetization, monetary authorities in Macedonia optimized liquidity in the banking sector in order to stabilize interest rate fluctuations. A properly estimated money demand function should help monetary authorities to determine the required bank reserves in line with future economic expectations (Tillers 2004).

The Money-in-the-utility-function (MIUF) model is derived in study 1 of this paper. One of the main results of the model is the property of superneutrality, implying that “the long-run capital stock of the economy is independent of the rate of monetary expansion” (Sidrauski 1967, p.s44). Given the fact that the patterns of real output and consumption are dependent of the real output and consumption are dependent of the rate of real output and consumption are dependent of the rate of monetary expansion” (Sidrauski 1967, p.544). Given the fact that the patterns of real output and consumption are dependent of the rate of real output and consumption are dependent of the rate of monetary expansion” (Sidrauski 1967, p.544). The property of superneutrality is derived under the assumption that consumption and money are separable in the utility function. In the case where the utility function is non-separable, the steady-state values of the variables depend on the nominal money growth through its effects on inflation. In the case of non-separability, the household’s decisions are affected by money balances, and nominal variables have real effects on the economy (Schabert and Stoltenberg 2005).

In the study 2, we derive the basic money demand equation and use the cointegration approach to test the long-run income and interest rate elasticity of the money demand in the case of the Republic of Macedonia. The majority of empirical works on the money demand functions were conducted on the case of developed countries (see for example Sriram 1999). Most of the empirical literature on developed countries represents the demand for money as a function of only two variables: aggregate income, usually represented by the real GDP, and interest rates as an approximation of the opportunity cost of holding money. It is very important to point out that this approach assumes the Fisher equation holds, and the rate of inflation is included in the money demand equation through the interest rates. However, this may not be the case with Macedonia, given the fact that the financial market is still underdeveloped and the capital mobility is limited. In order to address this issue, we followed the approach developed by Cziraky and Gillman (2006). First, we test whether the Fisher relationship holds in the case of Macedonia. If this relationship holds, the inflation rate is already included in the money demand equation through the opportunity cost variable. If the relationship doesn’t hold, we must include the rate of inflation in the money demand estimation. Next, we estimate the money demand equation represented by the M2 aggregate using the cointegration approach. The money demand estimation is presented in the study 2 of this paper.

STUDY 1: THE MONEY-IN-THE-UTILITY-FUNCTION

Literature review

The MIUF model presented in the next section derives two popular properties: the conditions of the long-run neutrality and superneutrality of money. The conditions of the long-run neutrality (LRN) and long-run superneutrality (LRSN) of money were empirically challenged in recent decades. Some of the earlier tests include Geweke (1986) and Stock and Watson (1989). Fisher and Seater (1993) analyzed these conditions in the bivariate ARIMA framework. This framework is highly sensitive to the order of integration: if the real variables are integrated of order 1, monetary variables should be integrated of order 2. Using this framework, some studies reported evidence in favor of the LRN conditions: Weber (1994) for the G-7 countries and Wallace (1999; 2005) for Mexico and Guatemala. Telatar and Cavusoglu (2005) analyzed the LRN condition in the case of five high-inflation countries. The results show that the LRN condition cannot be rejected in the cases of Brazil, Mexico and Turkey. The LRN condition is rejected in the cases of Argentina and Uruguay.

King and Watson (1992) provided another general framework to test the LRN condition, based on the VAR model. King and Watson (1992) criticized the Fisher-Seater framework and argued that it is subject to the Colley and LeRoy critique. Chen (2007) examined the LRN using a methodology developed by King and Watson (1992) for South Korea and Taiwan. Their results suggest strong support for the LRN condition in South Korea and only little evidence in the case of Taiwan.

The model: derivation and simulations

In the first section we derive the money in the utility function, a model introduced by Sidrauski (1967). Sidrauski (1967) assumed that real money holdings increase the welfare of economic agents and therefore that money can be incorporated directly into the household’s utility functions. Derivations of the model
are similar to Walsh (2010) and Wong (2013). In the MIUF model, households optimize their holdings or real money balances ($m_t$) and their consumptions paths ($c_t$) in order to maximize the present value of their utility function:

$$W = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t)$$

(1)

Where $\beta$ represents the discount rate and $0 < \beta < 1$, and $c_t$ and $m_t$ are consumption and real money holdings in per-capita terms. The household’s optimization problem is subject to the following budget constraints (Walsh 2010, p.36):

$$r_t N_t + (1 - \delta) K_{t-1} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} = C_t + K_t + \frac{M_t}{P_t} + \frac{B_t}{P_t}$$

(2)

Where $r_t$ represents the per-capita real lump-sum transfers from the government, $B_t$ denotes bond holdings, $i_t$ denotes the interest rate and $K_t$ denotes the stock of capital. If we divide by $N_t$ (population), we can rewrite the equation (2) in per-capita terms:

$$f \left( \frac{k_{t-1}}{1+n} \right) + r_t + \left( \frac{1-\delta}{1+n} \right) k_{t-1} + \frac{(1+i_{t-1})b_{t-1}+m_{t-1}}{(1+n)(1+\pi_t)} = c_t + k_t + m_t + b_t$$

(3)

- where $\pi_t$ represents the inflation rate. Equation (3) can be formulated as a “Bellman equation,” where the household’s problem is to maximize the value function $V$ by choosing optimal paths of $c_t, k_t, m_t$ and $b_t$.

$$V(W_t) = \max \{u(c_t, m_t) + \beta V(W_{t+1})\}$$

(4)

s.t:

$$W_{t+1} = f \left( \frac{k_{t+1}}{1+n} \right) + r_{t+1} + \left( \frac{1-\delta}{1+n} \right) k_{t+1} + \frac{(1+i_t)b_t+m_t}{(1+n)(1+\pi_{t+1})} = c_{t+1} + k_{t+1} + m_{t+1} + b_{t+1}$$

(5)

From Equation (5) we can rewrite for the capital ($k_t$) in per-capita terms as:

$$k_t = W_t - c_t - m_t - b_t$$

(6)

Now, we can put equation (6) and equation (5) into equation (4):

$$V(W_t) = \max \left\{ u(c_t, m_t) + \beta V(W_{t+1}) \left[ f \left( \frac{W_t - c_t - m_t - b_t}{1+n} \right) + r_{t+1} + \left( \frac{1-\delta}{1+n} \right) (W_t - c_t - m_t - b_t) \right] + \frac{(1+i_t)b_t+m_t}{(1+n)(1+\pi_{t+1})} \right\}$$

(7)

The representative agent has three control variables: real money balances, consumption and bond holdings. The three FOCs for the maximization problem are:

$$\frac{\partial V(W_t)}{\partial c_t} = 0$$

(8)

$$\frac{\partial V(W_t)}{\partial b_t} = 0$$

(9)

$$\frac{\partial V(W_t)}{\partial m_t} = 0$$

(10)

And the transversality condition is:

$$\lim_{t \to \infty} \beta^t \lambda_t x_t = 0, \text{for } x_t \in \{c_t, m_t, b_t\}$$

(11)

The FOC with respect to consumption are:

$$\frac{\partial V(W_t)}{\partial c_t} = u_c + \beta V'(W_{t+1}) \left[ f_{kr} \left( \frac{1}{1+n} \right) \right] - \frac{1-\delta}{1+n} = 0$$

(12)

The FOC with respect to bond holdings are:

$$\frac{\partial V(W_t)}{\partial b_t} = \beta V'(W_{t+1}) \left[ f_{kr} \left( \frac{1}{1+n} \right) \right] - \frac{1-\delta}{1+n} + \frac{(1+i_t)}{(1+n)(1+\pi_{t+1})} = 0$$

(12)
\[
\frac{\partial V(W_t)}{\partial \pi_t} = -f_{k_t'} -(1-\delta) + \frac{(1+i_t)}{1+\pi_{t+1}} = 0 \tag{13}
\]

The FOC with respect to real money holdings holdings are:

\[
\frac{\partial V(W_t)}{\partial m_t} = u_m + \beta V'(W_{t+1}) \left( f_{k_t'} - \frac{1}{1+n} \right) - \frac{(1-\delta)}{1+n} + \frac{1}{(1+n)(1+\pi_{t+1})} = 0
\]

\[
\frac{\partial V(W_t)}{\partial m_t} = u_m - \frac{\beta}{1+n} V'(W_{t+1}) \left( f_{k_t'} + 1 - \delta - \frac{1}{1+\pi_{t+1}} \right) = 0 \tag{14}
\]

Taking partial derivative with respect to \(W_t\) yields to:

\[
\frac{\partial v_t(W_t)}{\partial W_t} = \beta V'_t(W_{t+1}) \frac{\partial W_{t+1}}{\partial W_t} \tag{15}
\]

and

\[
\frac{\partial W_{t+1}}{\partial W_t} = f_{k_t', \frac{1}{1+n} + \frac{(1-\delta)}{1+n}} \tag{16}
\]

Now, equation (15) can be rewritten as:

\[
\frac{\partial v_t(W_t)}{\partial W_t} = \beta V'_t(W_{t+1}) \left[ f_{k_t', \frac{1}{1+n} + \frac{(1-\delta)}{1+n}} \right]
\]

\[
V'_t(W_t) = \frac{\beta}{1+n} (f_{k_t'} + 1 - \delta) V'_t(W_{t+1}) \tag{17}
\]

\[
V'_t(W_{t+1}) = V'_t(W_t) \left[ \frac{\beta}{1+n} (f_{k_t'} + 1 - \delta) \right]^{-1} \tag{18}
\]

Making use of the equation (18), equation (12) can be written as:

\[
u_c = V'(W_t) \tag{19}\]

And equation (14):

\[
u_m - \frac{\beta}{1+n} V'(W_{t+1})(f_{k_t'} + 1 - \delta) + \frac{\beta}{1+n} \frac{1}{1+\pi_{t+1}} V'(W_{t+1}) = 0 \tag{20}
\]

\[
u_m - u_c(c_t, m_t) + \frac{\beta}{1+n} \frac{1}{1+\pi_{t+1}} u_c(c_{t+1}, m_{t+1}) = 0
\]

\[
u_m + \frac{\beta}{1+n} \frac{1}{1+\pi_{t+1}} u_c(c_{t+1}, m_{t+1}) = u_c(c_t, m_t)
\]

\[
\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} + \frac{\beta}{1+n} \frac{1}{1+\pi_{t+1}} \frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} = 1
\]

\[
\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = 1 - \frac{1}{(1+n)(1+\pi_{t+1})} \beta \frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} = 1 \tag{21}
\]

Assuming that \(r_t = f_{k_t'} - \delta\) and making use of equation (17):

\[
u_c(c_t, m_t) = \frac{\beta}{1+n} (1+r) u_c(c_{t+1}, m_{t+1})
\]

\[
\frac{u_c(c_{t+1}, m_{t+1})}{u_c(c_t, m_t)} = \frac{1+n}{\beta (1+r)} \tag{22}
\]

Finally, we can rewrite equation (21) by plugging equation (22) into it:

\[
\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = 1 - \frac{1}{(1+n)(1+\pi_{t+1})} \beta \frac{1+n}{\beta (1+r)} = 1
\]

\[
\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i}{(1+i)} \equiv \omega \tag{23}
\]
Here, we assume that \((1 + i_t) = (1 + r_t)(1 + \pi_{t+1})\), or that the Fisher relationship holds. (Walsh 2010). Equation (23) is the key relation in the MIUF model: the intertemporal substitution between consumption today and tomorrow is a function of the interest rate. Finally, we have to assume the production function to be able to solve the model:

\[ y_t = e^{z_t} k_t^{\alpha} \]  

(24)

- where the productivity can be defined as an autoregressive (AR1) process:

\[ z_t = \rho z_{t-1} + \varepsilon_{zt} \]  

(25)

And \( \varepsilon_{zt} \) is a white noise technology shock. The steady state values of the variables and the set of log-linearized equations are presented in appendices 1 and 2.

**Calibration**

In order to simulate the main properties of the model, we use the “calibration” technique developed by Kydland and Prescott (1982). Following this technique, the values of the parameters are based on the averages of the aggregate data or on the previous microeconomic studies. In this paper, the values of the parameters are chosen to represent the characteristics of the Republic of Macedonia. We use the value of the gross capital formation as a percentage of GDP as approximation of the capital ratio (\( \alpha \)). The computed average share of gross capital formation in the real GDP in Macedonia for the period from 2002 to 2012 is 23,87% (table 1), so the value of the parameter \( \alpha \) is set to 0,2387. Following Petkovska (2008), the parameter for the rate of depreciation is set according to the amortization data for the Republic of Macedonia. The value of the rate of depreciation is set to 0,025 (quarterly) or 10% on annual basis, which is in accordance with the amortization data from the State Statistical Office of Macedonia (Petkovska 2008). Parameter \( \beta \) (subjective rate of discount) is based on the data for the average real interest rates in Macedonia. The real interest rates are estimated as the bank’s lending rates adjusted for the inflation rate. The subjective discount rate is set to 0,979 to match the average real interest rate of 8,25% (table 2). The value for parameter \( \rho_z \) (autoregressive parameter) is based on our estimations on the monetary aggregate M2, as an autoregressive process. The value of the autoregressive parameter \( \rho_z \) in the money growth process is set to 0,52 and the standard deviation is set to 0,046 based on our estimations for the M2 nominal money growth as an AR(1) process on a quarterly data from 2002 to 2012.

\[ \Delta LM2 = 0,52 \Delta LM2(-1) + \varepsilon_t \]

The remaining parameters are set according to Walsh (2010) and Brzoza-Brzezina (2011). We proceed with simulations of the MIUF model, estimated and simulated using the DYNARE platform developed by Michael Julliard.

**Simulations**

In this section, we analyze the simulated responses of the output, inflation and interest rate on productivity and money growth shocks. The simulated impulse responses reveal the main property of the money-in-the-utility function – the superneutrality of money. The responses of the variables to 1% positive innovation shocks in technology and nominal money supply are presented in the pictures 1 and 2, respectively.

Immediately after the shock in productivity, prices fall and output increases (picture 1). The simulations imply that the real variables (consumption, output and capital stock) are largely driven by productivity.

---

**Table 1:** Gross Capital formation in Macedonia as a share of GDP for the period 2002-2012.

<table>
<thead>
<tr>
<th>Year</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>20,6</td>
<td>19,1</td>
<td>22</td>
<td>21,3</td>
<td>21,5</td>
<td>24,6</td>
<td>26,8</td>
<td>26,2</td>
<td>24,9</td>
<td>26,2</td>
<td>29,4</td>
<td>23,87</td>
</tr>
</tbody>
</table>

*Source: NBRM.*

**Table 2:** Real interest rates (%) for the period 2002-2012

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>14,81</td>
<td>12,84</td>
<td>11,64</td>
<td>8,06</td>
<td>7,9</td>
<td>1,39</td>
<td>10,87</td>
<td>7,88</td>
<td>4,96</td>
<td>5,17</td>
<td>5,25</td>
</tr>
</tbody>
</table>

*Source: NBRM, International Financial Statistics and author’s calculations.*
shocks. On the other hand, the real variables (except real money balances) are unaffected by the growth rate in the nominal money supply and the inflation rate (Picture 2). The inflation and nominal interest rates rise immediately after the shock in the nominal money supply, but the patterns of consumption are unaffected by the changes in the rate of inflation (Picture 2). However, the positive correlation between the inflation and the nominal interest rates is inconsistent with the empirical findings of the “liquidity effect” of money on the interest rates: in the short-run there is a negative response in the interest rates on the shocks to monetary supply (Friedman 1968; Cagan 1972). Here, the simulations imply that the effects of “anticipated inflation” dominate the “liquidity effects.” The nominal interest rates rise following a positive money supply shock because of the higher anticipated inflation.

The property of superneutrality is derived under the assumption that consumption and money are separable in the utility function. In the case of non-separability, the steady-state values of the variables will depend on the nominal money growth through its effects on the rate of inflation. The issue of separability

---

**Picture 1: IRFs to a productivity shock**

![Picture 1: IRFs to a productivity shock](image)

**Picture 2: IRFs to a money supply shock**

![Picture 2: IRFs to a money supply shock](image)
has been empirically tested by some authors. Andrés, López-Salido and Vallés (2006) confirm the separability between the real balances and consumption in the case of the Euro Zone data. The results show that fluctuations in production and prices are largely driven by shocks in the real variables. Similar to Andrés, López-Salido and Vallés (2006), Ireland (2004) found only a limited effect from money on the fluctuations of output and prices in the case of the United States. The basic MIUF model has been extended to introduce the property of non-supernaturality - the growth rate of the nominal money and the inflation rate have real effects on the steady state values of real variables (Walsh 2010). One way to generate this property is to add a labor-leisure choice, while the other is to add non-separable preferences (see for example Heer 2004; Schabert and Stoltenberg 2005).

**STUDY 2: MONEY DEMAND ESTIMATION**

During the last two decades, the role of the monetary aggregates in the monetary policy research has been strongly reduced. Many studies, particularly those made in the case of developed countries, focused on interest rates as the main channel of the transmission mechanism of monetary policy. However, long-run aggregate economic activity and inflation dynamics are affected by the disequilibrium in the demand for money. The disequilibrium in the demand for money may affect the efficiency of the interest rates as the most important channel of monetary policy in developed countries (Valadkhani and Alludin 2003). This implies that interest rates can be indirectly affected by money demand through its effect on the output gap.

This paper applies the Vector error-correction model to identify both short-run and long-run-equilibrium dynamics between the M2 aggregate and its determinants. In this stage, a properly specified money demand function is of critical importance for the validity of the estimation process. In specifying the money demand function, we follow the approach developed by Cziráky and Gillman (2006). Following Cziráky and Gillman (2006), we test whether the Fisher equation holds in the case of the Republic of Macedonia. If the Fisher relation holds, the inflation rate is already included in the money demand equation through the interest rates. If the Fisher relation doesn't hold, we must include inflation in the money demand specification.

**Literature review**

The majority of empirical works on money demand functions use cases of developed countries. However, during the last two decades the money demand function was also estimated for Central and Eastern European countries.


There have been studies of money demand functions in the case of Macedonia, mainly focused on the monetary aggregate M1. Petrovski and Jovanovski (2010) analyzed the long-run and short-run dynamics of the money demand function in the case of Macedonia using the VEC model. They use the M1 monetary aggregate as an approximation of money, and a quarterly frequency over the period from 1994 to 2008. The results suggest a stable demand for money over the analyzed period. The estimated long-run income elasticity is lower than unity (0.64) and high semi-elasticity with respect to the interest rate. Kjosevski (2013) used a VEC model and monthly data over the period from 2005 to 2012. Similar to Petrovski and Jovanovski (2010), Kjosevski finds a stable demand for M1, long-run income elasticity lower than unity, and slow adjustment to the equilibrium. However, Kjosevski (2013) finds a significantly lower interest elasticity (-0.25), but we should note that the authors estimated the demand for money over two different periods.

In this paper, we will examine the demand for money on the case of Macedonia as represented by the broader monetary aggregate M2.
**Data and modelling**

**Variable selection and data transformations**

The selection of appropriate variables to be included in the money demand estimation is of critical importance. Sriram (1999) points out that the selection of opportunity cost variables is the main factor for the differences of the estimated money demand functions.

To estimate the money demand function in the case of Macedonia, we use quarterly data over the period from 2002 to 2012, taken from the *International Financial Statistics database*. The quarterly time series are: the M2 monetary aggregate (nominal); the consumer price index (CPI); deposit interest rates and real GDP. The series for the rate of inflation are constructed as a first difference of the consumer price index.

The time series for the real GDP, M2 and CPI were seasonally adjusted using the Census X12 filter available in eViews7 software. All data except the deposit interest rates were transformed as a natural logarithm. In order to properly estimate the money demand function, we must transform the nominal M2 into real M2. The consumer price index (CPI) is used to transform the nominal money balances (M2 aggregate) into real money balances (M2 adjusted for the inflation - M2/CPI). Contrary to the previous studies in the case of Macedonia, here we use a broader measure of money (M2). According to Valadkhani and Alludin (2003), the M2 monetary aggregate is less distorted by the process of financial innovation and has a closer relationship with the measures of economic activity. The M2 monetary aggregate “includes the monetary aggregate M1 and short-term deposits” (NBRM 2013, p.18). Deposit interest rates represent the interest rates paid by commercial banks on demand, time, or savings deposits. Using the long-term interest rates seems appropriate for the broader monetary aggregates, in order to capture financial asset substitutions (Valadkhani and Alludin 2003). Real GDP series are used to represent the aggregate income (Y) and deposit interest rates and the rates of inflation are used as proxies for the opportunity cost of holding money (i). The Real GDP variable represents the transaction or wealth effects and economic theory predicts a positive relationship between money and output (Sriram 1999). At the same time, economic theory predicts a negative relationship between money and the variables representing the opportunity costs of holding money. As we mentioned earlier, we included the rate of inflation after testing the validity of the Fisher equation.

Some authors also use the exchange rate in the money demand specification to capture the effects from the substitution between domestic and foreign money (Dobnik 2011). However, we excluded the exchange rate from the money demand specification given the fact that the denar-euro exchange rate remained fixed for the analyzed period.

The Augmented Dickey-Fuller (1979) test was carried out to test for unit root. The ADF test shows that all variables except the inflation rate are integrated of order (1) (Table 3).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Level (t-stat)</th>
<th>First diff. (t-stat)</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td>real money balances</td>
<td>m – p</td>
<td>-2.2228</td>
<td>-5.2708</td>
<td>I(1)</td>
</tr>
<tr>
<td>cpi</td>
<td>P</td>
<td>1.1090</td>
<td>-3.4622</td>
<td>I(1)</td>
</tr>
<tr>
<td>deposit interest rate</td>
<td>R</td>
<td>-2.3592</td>
<td>-4.6022</td>
<td>I(1)</td>
</tr>
<tr>
<td>inflation</td>
<td>II</td>
<td>-2.6671</td>
<td>-4.1273</td>
<td>I(1)</td>
</tr>
<tr>
<td>real GDP</td>
<td>Y</td>
<td>-0.9222</td>
<td>-9.0900</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

**The Fisher equation**

In order to specify the money demand equation, we follow the approach developed by Cziraky and Gillman (2006). First, we need to test the validity of the Fisher equation in the case of Macedonia. If the Fisher equation holds, the inflation rate is included in the money demand equation through the nominal interest rate (Dreger, Reimers and Roffia 2007). In this case, the inflation rate should be excluded from the money demand equation. If the Fisher equation does not hold, we need to include the inflation rate in the money demand function. The Fisher equation represents the relationship between nominal and real interest rates, and it was derived in the money-in-the-utility function model (Study 1). The fisher equation can be written as: (Cziraky and Gillman 2006)

\[ \rho_t = \alpha + \hat{\epsilon}_t \]  

where \( \rho \) represents the nominal interest rate, \( \alpha \) represents the real interest rate and \( \pi \) represents the inflation rate. We assume that the real interest rate equals:

\[ \rho_t = \alpha + \hat{\epsilon}_t \]  

Where \( \alpha \) term is a constant and \( \hat{\epsilon}_t \) is a white noise process. Following Cziraky and Gillman (2006) the Fisher equation can be rewritten as:

\[ r_t = \alpha + \pi_t + \hat{\epsilon}_t \]  

(t-stat)result

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Level (t-stat)</th>
<th>First diff. (t-stat)</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td>real money balances</td>
<td>m – p</td>
<td>-2.2228</td>
<td>-5.2708</td>
<td>I(1)</td>
</tr>
<tr>
<td>cpi</td>
<td>P</td>
<td>1.1090</td>
<td>-3.4622</td>
<td>I(1)</td>
</tr>
<tr>
<td>deposit interest rate</td>
<td>R</td>
<td>-2.3592</td>
<td>-4.6022</td>
<td>I(1)</td>
</tr>
<tr>
<td>inflation</td>
<td>II</td>
<td>-2.6671</td>
<td>-4.1273</td>
<td>I(1)</td>
</tr>
<tr>
<td>real GDP</td>
<td>Y</td>
<td>-0.9222</td>
<td>-9.0900</td>
<td>I(1)</td>
</tr>
</tbody>
</table>
The empirical estimation of the validity of Fisher effect can be presented as:

$$\ln(r_t) = \beta_0 + \beta_1 \ln(\pi_t) + \varepsilon_t$$  \hspace{1cm} (37)

where $r_t$ is represented by the deposit interest rates and $\pi_t$ is represented by the rate of inflation. To test for cointegration, here we apply the Engle-Granger two-step procedure. First, we need to estimate the Fisher effect and then we will perform an Augmented Dickey-Fuller (ADF) test on the residuals from the estimated Fisher equation. The estimated Fisher equation can be presented as:

$$\ln(r_t) = 6.68 - 0.13 \ln(\pi_t) + \varepsilon_t$$

The ADF test on the residuals from the estimated Fisher equation suggests that the residuals are integrated of order 1 (Table 4). Since the residuals are non-stationary, the results imply that the Fisher equation does not hold in the Republic of Macedonia. Based on the results, following Cziraky and Gillman (2006), we will include the inflation rate in the specification of the long-run money demand function.

Table 4: Engle-Granger test for cointegration

<table>
<thead>
<tr>
<th>Lag Order Selection Criteria (Unrestricted VAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Money demand estimation: results and discussion

We employ a VEC model to capture the long-run and short-run interactions between M2 and its determinants. The long-run money demand equation can be specified as:

$$(m - p) = \alpha y + \beta r + \psi \pi$$  \hspace{1cm} (38)

where $\alpha$ represents income elasticity and $\beta$ represents the semi-elasticity of money demand with respect to deposit interest rate.

To determine the appropriate lag length to be included in the test for the rank of cointegration, we carried out a lag length test on the unrestricted VAR model (Enders 2009). Here, we choose the two lags based on the Final prediction error (FPE) criteria (Table 5).

Table 5: VAR Lag Order Selection Criteria

The cointegration approach requires all variables to be integrated in the same order (Hafer and Jansen 1991, Enders 2009, Brooks 2002). In our case, all of the variables included in the money demand specification are integrated of order 1. The Johansen test rejects the hypothesis of no cointegration. The results from the Johansen test indicate two cointegration vectors based on the Trace test and one cointegration vector based on the Maximum Eigenvalue test (Table 6). We proceed with the estimation of Vector Error Correction model based on one cointegration equation.

Table 6: Johansen Cointegration test

Money-in-the-utility-function: Model simulations and money demand estimation in the case of the Republic of Macedonia
Money-in-the-utility-function: Model simulations and money demand estimation in the case of the Republic of Macedonia

1992). Also, we use a broader measure of money demand (M2 aggregate) which we believe is a better definition of money. According to the quantity theory of money, an income elasticity lower than unity implies the increasing velocity of money. However, these findings are inconsistent with the data for the Republic of Macedonia, which suggests a declining velocity of money (Appendix 4). As Judd and Motley (1984) suggest, as economic agents become more responsive to interest rates, money demand reacts more aggressively to the changes in the interest rates, leading to the declining velocity of money. The trend of declining velocity is also consistent with the process of financial and economic development (Chowdhury 1994). As Chowdhury (1994) pointed out, the velocity should decline at a slower rate at higher levels of economic development. This implies that estimated income elasticity should decline in the future. The estimated semi-elasticity with respect to deposit interest rates is small and negative (-0.17), indicating that economic agents are willing to decrease their money holdings when opportunity costs rise. Although the estimated coefficient carries the expected sign, it is statistically insignificant at 5%, and almost insignificant at 10% (p=0.11). Compared to previous studies (Petrevski and Jovanovski 2010, Kjosevski 2013), the estimated interest elasticity for M2 is lower. According to Nell (1999), interest elasticity for the broader monetary aggregates is likely to be much smaller than those for narrow money. The results indicate negative elasticity with respect to inflation rate (-0.38), implying that economic agents decrease their money holdings less than proportionally as the inflation rises. The coefficient before the inflation rate is significant at 1%.

The short-run dynamics reveal that only 2.70% of the disequilibrium is corrected in a single quarter. The expected negative sign on the error-correction term is highly significant. This relatively slow adjustment to the equilibrium level is consistent with the previous findings based on M1. The short-run dynamics are presented in the Appendix 3.

Finally, we perform diagnostic tests on the residuals from the estimated model. The results are presented in Appendix 5 and show that the model is correctly specified.

CONCLUSION

The main purpose of this paper was to reassess and analyze the dynamic interactions between money, prices and output in the case of Macedonia. In the first study, we derived and simulated Sidrauski’s MIUF model, calibrated to match the data for the Republic of Macedonia. The simulated impulse responses reveal the main property of this model - the superneutrality of money. The real variables were mostly driven by productivity shocks, whereas the monetary shocks didn’t have any real effects. Sidrauski’s (1967) model is mostly used to analyze the determinants of the money demand function. In the second study, we estimated the money demand function in the case of the Republic of Macedonia using VECM on quarterly data from 2002 to 2012. Following Cziráky and Gillman (2006) we tested the validity of the Fisher effect to decide whether to include the inflation rate in the money demand equation. The Fisher effect does not hold in the case of Macedonia, so we included the inflation rate in the money demand equation. We found one cointegrating vector between the real money balances, deposit interest rates, rate of inflation and the scale variable using Johansen’s multivariate approach. The money demand equation was estimated using a vector error-correction framework. The results of the cointegration equation (normalized to real money balances) are in line with economic theory. The estimated income elasticity is less than unity (0.81) and consistent with previous studies in the case of Macedonia. However, we must note that previous studies were focused on the monetary aggregate M1 rather than M2. Also, previous studies covered different periods. The estimated semi-elasticity of money demand with respect to the interest rate is small and negative (-0.17), suggesting that economic agents are willing to decrease their real money holdings when opportunity costs rise. The estimated interest rate of semi-elasticity reported here is smaller compared to the coefficient reported by Petrevski and Jovanovski (2013) and Kjosevski (2013). However, as Nell (1999) argued, interest rate elasticities for broader monetary aggregates are likely to be much smaller than those for narrow money. Finally, the results indicate negative elasticity with respect to the inflation rate (-0.38) implying that economic agents decrease their money holdings less than proportionally as inflation rises. The short-run dynamics reveal that only 2.70% of the disequilibrium is corrected in a single quarter. The properties of stability imply that the M2 aggregate may serve as a proper policy indicator. The estimated money demand function should help monetary authorities to optimize liquidity in the banking sector, in accordance with economic expectations.
REFERENCES


APPENDIX

1. Steady state

In this section, we define the steady state values of the variables in the model. The steady state values are the values of consumption, inflation, interest rate, money balances and capital stock to which the economy converges in the absence of shocks (Walsh 2010; Brzoza-Brzezina 2011). There are two important assumptions in the MIUF model:
- 0% population growth (n=0), and
- 0% productivity growth.

The nominal money supply can be expressed as a simple stochastic process:

\[ M_t = e^{\theta_t} M_{t-1} \]  
(26)

And the real money supply in per-capita terms can be written as:

\[ m_t = \frac{m_{t-1}}{n_t} e^{\theta_t} \]  
(27)

\[ \theta_t = \rho \theta_{t-1} + \epsilon_{\theta,t} \]

- where \( \epsilon_{\theta,t} \) represents the money supply shock.

Also, the MIUF model assumes zero net supply of government bonds \( b_t = 0 \), and the government budget is balanced every year (Brzoza-Brzezina 2011). This relation implies that monetary policy is not independent of fiscal policy.

\[ r_t + n_t = M_t - M_{t-1} \]  
(28)

or in per-capita terms:

\[ r_t = m_t - \frac{m_{t-1}}{n_t} \]  
(29)

If we use equation (29), we can rewrite the budget constraints as:

\[ y_t + (1 - \delta)k_{t-1} = c_t + k_t \]  
(30)

The assumption of no productivity and polulation growth leads to the following steady state equations:

\[ \alpha(k_{ss})^{1-\alpha} = \frac{1}{\beta} - 1 + \delta \]  
(31)

\[ (k_{ss})^{1-\alpha} = \left( \frac{\alpha \beta}{1 + \beta(\delta - 1)} \right)^{\frac{1}{1-\alpha}} \]  
(32)

Equation (32) implies that steady state capital depends only on the discount rate, the rate of depreciation and on the production function (Walsh 2010). Putting equation (30) into production function (24) yields:

\[ c_{ss} = f(k_{ss})^\alpha - \delta k_{ss} \]

Now, we can use equation (32) to rewrite the previous equation as:

\[ c_{ss} = \left( \frac{\alpha \beta}{1 + \beta(\delta - 1)} \right)^{\frac{1}{1-\alpha}} - \delta \left( \frac{\alpha \beta}{1 + \beta(\delta - 1)} \right)^{\frac{1}{1-\alpha}} \]  
(33)

- where \( f(k) = k^\alpha \). Again, steady state consumption is independent of the rate of inflation and the rate of growth of the nominal money supply. Since the steady states of output, consumption and capitul per-capita are all independent of the growth rate of the nominal money supply and thus in the rate of inflation, this condition is commonly known as the superneutrality of money (Walsh 2010).

2. Log-linearization around the steady state and the money demand function

The model we derived in the previous section is non-linear, and non-linear models are difficult to solve. In order to be able to solve the model, we will use first-order Taylor approximations around the steady state (Uhlig 1995). The variables are described as percentage deviations around the steady state, and the model can be solved and simulated using the DYNARE platform developed by Michael Julliard.

Log-linearizing around the steady state leads to the following system of equations (Walsh, 2010):

\[ \hat{y}_t = \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t + z_t \]

\[ \frac{y_{ss}}{k_{ss}} \hat{y}_t = \frac{y_{cs}}{k_{cs}} \hat{c}_t + \hat{k}_t + (1 - \delta) \hat{k}_{t-1} \]

\[ E_t[\Omega_1 (\hat{c}_{t-1} - \hat{c}_t) + \Omega_2 (\hat{m}_{t-1} - \hat{m}_t)] - \hat{n}_t = 0 \]

\[ [\hat{y}_t - \Omega_1 \hat{c}_t + \Omega_2 \hat{m}_t] = \left( 1 + \frac{n_{ss}}{1 - n_{ss}} \right) \hat{n}_t \]

1 Full derivation of the log-linearized equations can be found in Walsh (2010). In this paper, we simplify the model and the dynamics of employment are excluded from the model.
\[ \hat{r}_t = a \frac{\gamma^{ss}}{k^{ss}} (E_t \hat{y}_{t+1} - \hat{k}_t) \]
\[ \hat{i}_t = \hat{r}_t + E_t \hat{r}_{t+1} \]
\[ \hat{m}_t = \hat{M}_t - \hat{p}_t = \hat{c}_t - \left( \frac{1}{b} \right) \hat{i}_t \]
\[ \hat{m}_t = \hat{m}_{t-1} - \hat{h}_t + u_t \]
\[ \hat{y}_t = \alpha \hat{k}_{t-1} + z_t \]
\[ \hat{m}_t = \hat{m}_{t-1} + \Omega - \hat{h}_t + u_t \]
\[ \hat{c}_t = E_t (\hat{c}_{t+1} + \hat{r}_t) \]
\[ \hat{i}_t = \hat{r}_t + E_t \hat{r}_{t+1} \]
\[ \hat{m}_t = \hat{c}_t - \left( \frac{1}{i^{ss} - 1} \right) \hat{i}_t \]
\[ r^{ss} \hat{r}_t = a \frac{\gamma^{ss}}{k^{ss}} E_t (\hat{y}_{t+1} - \hat{k}) \]
\[ \hat{k}_t = (1 - \delta) \hat{k}_{t-1} + \frac{\gamma^{ss}}{k^{ss}} \hat{y}_t - \frac{\gamma c^{ss}}{k^{ss}} \hat{c}_t \]
\[ z_t = \rho z z_{t-1} + \epsilon_t \]
\[ \Omega = \rho \Omega \Omega_{t-1} + \mu_t \]

The parameters \( r^{ss}, i^{ss} \) and \( \pi^{ss} \) are defined in the following way:
\[ r^{ss} = \frac{1}{\beta} \]
\[ \pi^{ss} = 1 \]
\[ i^{ss} = \pi^{ss} r^{ss} = r^{ss} \]

4. M2 money stock velocity in the Republic of Macedonia:

Graph 1: M2 money stock velocity

Source: Author’s calculations based on IFS data.

5. VECM Diagnostic tests:

<table>
<thead>
<tr>
<th>Lags</th>
<th>Q-Stat</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>18,34313</td>
<td>0,9172</td>
</tr>
<tr>
<td>4</td>
<td>42,06867</td>
<td>0,5547</td>
</tr>
<tr>
<td>5</td>
<td>55,18934</td>
<td>0,6518</td>
</tr>
<tr>
<td>6</td>
<td>64,18571</td>
<td>0,8310</td>
</tr>
<tr>
<td>7</td>
<td>78,18226</td>
<td>0,8473</td>
</tr>
<tr>
<td>8</td>
<td>104,4223</td>
<td>0,5795</td>
</tr>
<tr>
<td>9</td>
<td>113,4382</td>
<td>0,7415</td>
</tr>
<tr>
<td>10</td>
<td>124,7172</td>
<td>0,8182</td>
</tr>
<tr>
<td>11</td>
<td>136,4046</td>
<td>0,8689</td>
</tr>
<tr>
<td>12</td>
<td>142,8652</td>
<td>0,9488</td>
</tr>
</tbody>
</table>

Table 9: VEC Residual Heteroskedasticity Tests: No Cross Terms (only levels and squares):

<table>
<thead>
<tr>
<th>Component</th>
<th>Jarque-Bera</th>
<th>df</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,791174</td>
<td>2</td>
<td>0,4084</td>
</tr>
<tr>
<td>2</td>
<td>0,528712</td>
<td>2</td>
<td>0,7677</td>
</tr>
<tr>
<td>3</td>
<td>1,913792</td>
<td>2</td>
<td>0,3841</td>
</tr>
<tr>
<td>4</td>
<td>0,688563</td>
<td>2</td>
<td>0,7087</td>
</tr>
<tr>
<td>Joint</td>
<td>4,922242</td>
<td>8</td>
<td>0,7659</td>
</tr>
</tbody>
</table>

3. VECM: Short Run Dynamics

<table>
<thead>
<tr>
<th>Short-run dynamics (VECM)</th>
<th>coefficient</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM(-1)</td>
<td>-0,0269</td>
<td>0,0018</td>
</tr>
<tr>
<td>Δrealm2(-1)</td>
<td>0,1578</td>
<td>0,3599</td>
</tr>
<tr>
<td>Δrealm2(-2)</td>
<td>0,3487</td>
<td>0,055</td>
</tr>
<tr>
<td>Δincome(-1)</td>
<td>-0,3756</td>
<td>0,1708</td>
</tr>
<tr>
<td>Δincome(-2)</td>
<td>-0,2917</td>
<td>0,2915</td>
</tr>
<tr>
<td>Δdeposit(-1)</td>
<td>0,0012</td>
<td>0,9523</td>
</tr>
<tr>
<td>Δdeposit(-2)</td>
<td>0,0391</td>
<td>0,0419</td>
</tr>
<tr>
<td>ΔDinflation(-1)</td>
<td>0,0014</td>
<td>0,8014</td>
</tr>
<tr>
<td>ΔDinflation(-2)</td>
<td>0,0113</td>
<td>0,0514</td>
</tr>
</tbody>
</table>