

# MEASURING THE NATURAL INTEREST RATE FOR THE MACEDONIAN ECONOMY: A MULTI-MODEL APPROACH

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## Abstract

*This paper identifies the natural interest rate for the Macedonian economy using quarterly data for 2001Q4-2019Q3. To this end, the estimation is made by using different types of models, such as the Holston, Laubach, and Williams model and the full-fledged country-specific structural MAKPAM model. The empirical results show that the natural rate of interest in the Macedonian economy has declined over time, which is similar to the findings for other countries. The decomposition of the natural rate suggests that the main driver for the decline is the slowdown of the Macedonian potential GDP growth in the period after the global economic crisis, although there are signs of its recovery at the end of the sample period. In addition, the results indicate that the monetary policy conditions in the Macedonian economy have been broadly accommodative from 2011Q4 onwards. The substantive conclusions are unchanged across the multiple models used in this study.*

**JEL Classification:** C32, E43, E52, O40

**Keywords:** Natural interest rate, interest rate gap, Holston, Laubach, and Williams model, MAKPAM Model

## 1. Introduction

Many advanced economies have had low inflation now for an extended period, while the interest rates in some have reached record low levels. Additionally, the recent global economic slowdown has brought the need to introduce unconventional policy measures and ease the monetary policy by pushing the nominal interest rate into negative territory. This decline in the nominal and real interest rates has renewed the interest in determining the natural rate of interest. The natural rate of interest was first introduced by Wicksell as, “[the real] rate of interest on loans which is neutral

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in respect to commodity prices, and tends neither to raise nor to lower them" (Wicksell 1936, p. 102). A more recent definition of the natural rate of interest (or the equilibrium real rate) is the real interest rate level consistent with stable inflation, implicitly at the inflation target and with output at its equilibrium level (Hlédik and Vlček 2018).

In many advanced countries, where the short-term interest rate has become the primary policy instrument, the "equilibrium" or "natural" interest rate provides a metric for the policy stance. Therefore, the proper estimation of the natural rate of interest is crucial. So, the question arises whether the natural rate declined or does it remain at the pre-crisis levels? If the natural rate has declined, central bankers will have to keep their nominal interest rates below the pre-crisis level, otherwise, their monetary policy will be too restrictive, and conversely, if the natural rate remains at the pre-crisis level, monetary policy might be too expansionary. Additionally, the natural rate of interest is also important because the difference between the real interest rate and the natural rate – the real interest rate gap – is commonly used to measure the effects of monetary policy on the real economy. Having a short-term real rate below its natural level means, the monetary policy is stimulating the economy by creating a positive output gap, with an appropriate effect on inflation as well.

Therefore, the aim of this paper is to estimate the natural interest rate for the Macedonian economy, which would help in making more effective monetary policy decisions. For this purpose, we estimate the natural rate of interest based on several methodologies, including the Holston, Laubach, and Williams (2017) methodology and the modified Macedonian Policy Analysis Model (MAKPAM) model. In doing so, we ascertain the linkage between the natural interest rate, the potential output, and its trend growth and examine the empirical relationship between these unobserved variables. To the best of our knowledge, this is the first formal attempt to estimate the natural rate of interest for the Macedonian economy. Hence, this paper has an important contribution to enriching the empirical literature on this topic and is the first one related to the specifics of the topic, which has not been explored before in our country.

The paper is organized as follows. The second section briefly discusses the literature on the natural rate of interest, with a focus on the relevant empirical literature. The third section explains the data used and presents some brief stylized facts. The fourth section refers to the selected empirical methodology and discusses the main findings. The fifth section provides

some robustness checks. Conclusions are presented in the sixth section.

## 2. Related literature

One of the first seminal papers in the area of natural interest rate estimation was written by Laubach and Williams (2003). Through the application of the Kalman filter in a closed economy (the USA) setup, the authors estimated the natural rate of interest, potential output, and its trend growth, further examining the empirical relationship between these estimated series. Their results indicated that the US natural interest rate experienced a significant variation in the past forty years, with this variation being in a one-for-one relation with changes in the trend growth rate. Consequently, concluding that the trend growth rate was an important determinant of changes in the natural rate with time variation in the natural rate having important implications for the design and implementation of monetary policy. Therefore, adjustment to changes in the natural rate was crucial for the achievement of long-run inflation and short-run stabilization goals.

The Laubach and Williams (2003) method was also used for the case of the Czech Republic by Hlédik and Vlček (2018), but mainly modified in two aspects. First, the natural rate of interest was linked to equilibrium GDP growth, which was adjusted for real exchange rate appreciation (to incorporate the effect of koruna appreciation). Second, a semi-structural model closed by a monetary policy rule was employed, which allowed for forward-looking model-consistent expectations and imposed a comprehensive set of restrictions, i.e., model equations, to identify the natural rate of interest. Their estimations indicated that the natural rate of interest in the Czech economy in 2017 was close to 1 percent, a level lower compared to the 2015 peak, which was mainly driven by the appreciation of the equilibrium real exchange rate despite a recovery in the real economic activity.

On the other hand, the natural interest rate of the Danish economy was estimated by Pedersen (2015) using an empirical model adopted by Berger and Kempa (2014), instead of Laubach and Williams (2003), which according to him, is inappropriate due to the idiosyncratic characteristics of the Danish economy. Hence, the natural rate of interest was inferred from movements in GDP, applying a dynamic IS-relation and a Phillips-curve adopted from the small standard dynamic macroeconomic model theory. Pedersen (2015) found evidence of very low and perhaps negative levels of the natural real rate for Denmark, arguing

on an increased probability for these levels to remain in the near future and for the monetary policy rate to end up in the lower bound again.

Mendoca (2017) used the Laubach Williams (2003) methodology to estimate the natural rate of interest of Italy and the Netherlands. In the paper, the author detected a decreasing trend in the natural interest rates, further stating that the 'Wicksellian' rate of interest in these economies had reached a negative point. According to him, there was a link between the real interest rate gap and the output gap, but the natural interest rate estimates are subject to a high degree of uncertainty.

An interesting perspective on the natural rate of interest was given by Armelius, Solberger, and Spanberg (2017). The authors, besides estimating the Swedish natural rate of interest, also analyzed the determinants of its movement. Using a version of the Laubach Williams (2003) approach, they observe a declining trend in the Swedish natural rate in the last two decades, which had reached a negative value in 2017, with this decline being mainly due to unobserved variables and unrelated to the growth of potential GDP. Notably, they documented a significant influence from the US natural rate on the Swedish counterpart, as well as a separation in the real rate (analyzing its effect on house prices) in the natural rate part and a monetary policy part, with the influence of the latter getting stronger while the influence of the natural part close to zero.

Lastly, the Laubach and Williams (2003) approach was used by Manrique and Marques (2004) for the estimation of the US and German natural rates of

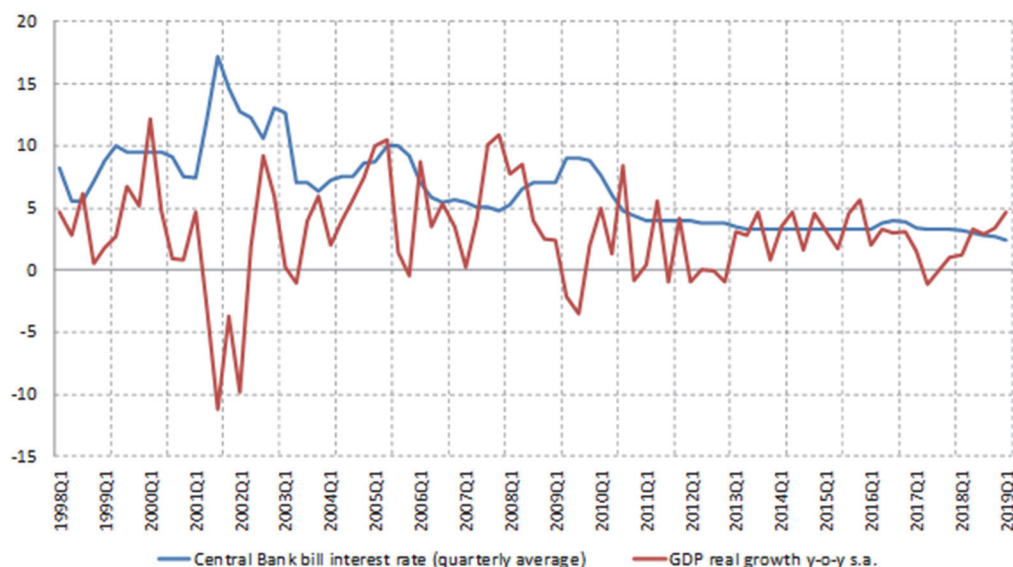
interest. Using quarterly data for the period from 1962 until 2001, the authors found that during the 90's the natural rate of interest and the output gap in both countries had shown volatility below their historical mean. Accordingly, the US natural rate of interest was situated between 1.6% and 2.3% in 2003 compared with the -0.6% of ex-post US real rate of interest, while in Germany it had fluctuated around 0.5% and 1.7%, closer to the observed real rate (0.3%) in this country.

### 3. Data and some stylized facts

We use quarterly data for a sample period from 2001 Q4 to 2019 Q3 (first estimate) to identify the Macedonian natural rate of interest. Data on domestic GDP at constant prices and inflation were obtained from State Statistical Office and data on foreign GDPs from Eurostat. The central bank bills rate of NBRNM was used as a measure of the actual nominal interest rate.

Figure 1 shows that the nominal interest rate on Central Bank bills had its highest levels during the years 2001 and 2002, reaching its peak of almost 18% in August 2001. The restrictive monetary policy during this period was taken to maintain macroeconomic stability during the times of the security crisis in the country, which severely affected the economy, resulting in deposit withdrawals, increased demand for foreign currency, decreased foreign exchange reserves, and pressures on the fixed exchange rate<sup>1</sup>. Additional contractionary measures were also adopted, among which was the increase of the reserve requirement

**Figure 1. Central Bank bill interest rate and real GDP growth**



rate and the Lombard rate, which helped to reduce the inflationary pressures and maintain price stability. The worsened security situation had a negative effect on the GDP, which experienced negative growth during this period. In the later period, during the pre-global financial crisis, the economy noted its highest real GDP growth with the referent nominal interest rates gradually falling. However, interest rates again started rising after the eruption of the global financial crisis in 2008. The main signs of the crisis included falling prices and a substantial fall in exports and private transfers. In such conditions, the decline of foreign exchange reserves again required a more restrictive monetary policy to support the sustainability of the fixed exchange rate regime, despite the absence of inflation pressures. After 2009, the nominal interest rate of the Central Bank bills was gradually reduced, reaching its lowest level of 2,25% in March 2019, and remaining at that level throughout the rest of the year. On the other hand, the GDP on average during this period has registered positive growth, with short episodes of stagnation in 2012 and 2017 as a result of the Eurozone debt crisis spillover effect and the escalation of the domestic political crisis.

Figure 2 shows the movement of inflation and the Central Bank bill interest rate. Overall, we can see that the inflation rate is relatively low and stable. However, exceptions are the periods in which there is a surge in the import prices of the primary commodities on the world market, which then swiftly translate into higher domestic prices. Furthermore, as mentioned before, if there are pressures on the foreign exchange rate, the central bank reacts by increasing the policy rate,

which can happen even in times of low inflation, such as in 2009. The most interesting is the period after 2013, when we see relatively low inflation, despite the gradual lowering of the nominal policy interest rate and relatively solid GDP growth, which is opposite to the suggestions from economic theory. Admittedly, for quite some time now, the inflation rate has remained below the historical average of 2%.

## 4. Specification of the models and empirical results

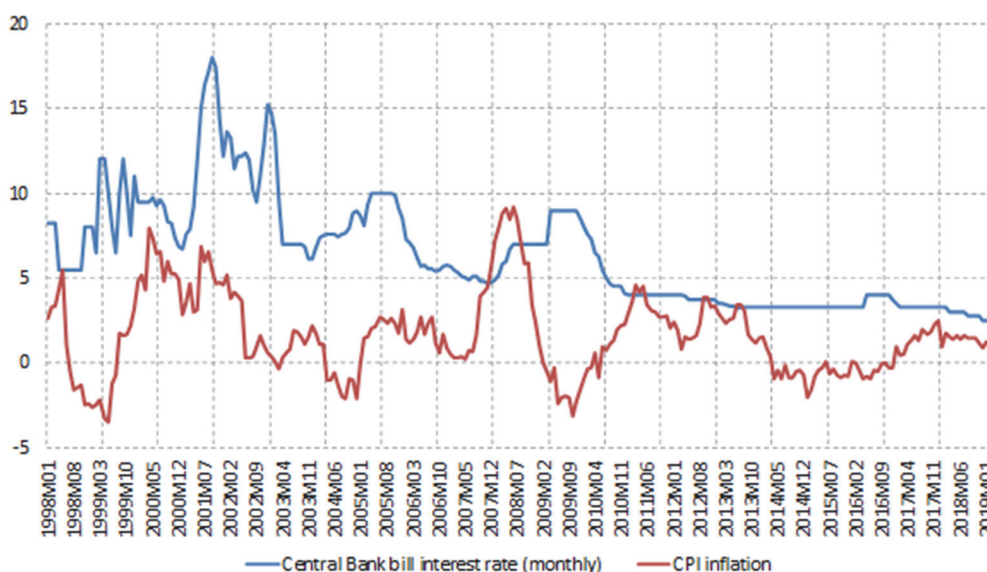
Given that the natural rate of interest is a latent variable, i.e. it cannot be directly observed, and its estimates are sensitive to the chosen methodology, we have opted to apply different model specifications to identify the natural rate of interest in the Macedonian case.

### 4.1. Holston-Laubach-Williams model

This section closely follows Scheerová (2017) and Fiorentini et al. (2018). Those are studies that rather systematically describe the technical characteristics and the very intuition behind the HLW (2017) methodology. Hence, providing enough detail necessary for the operationalization of the concepts established by the original research of HLW (2017).

Holston, Laubach, and Williams (2017) developed a methodology that offers a natural rate of interest evolution compatible with production structural

Figure 2. Central Bank Bills interest rate and annual inflation rate



changes, thus proposing a model structure that links the long-term movement in equilibrium real interest rate with the real characteristics of the economy. To this end, the real natural rate is being affected by both, the supply and demand-side structural factors. The supply side involves the potential growth of the economy (determined primarily by technology and factor endowments). The demand side is linked to demographical shifts, population growth, and the effects of aging on the elasticity of substitution and time preference (Scheerová 2017).

The equilibrium real interest rate is the rate that keeps the economy on a sustainable growth trajectory. In effect, it is the interest rate that allows the economy to grow in line with its potential and sustains full employment, while also ensuring that inflation meets the central bank's price stability target. It can also be thought of as the rate that balances planned investment and desired saving in an economy at full employment. In the long run, the rate is impacted by the amount of supply in the economy and is thus linked to potential GDP growth. Higher economic growth implies a higher return on capital, which in turn leads to stronger investment demand. In parallel, because it raises upcoming earnings, it guides forward-looking households to consume more and save less. Jointly, these outturns increase the natural real interest rate. The long-run equilibrium interest rate is not a directly measurable variable. That being the case, one has to infer it by considering the development of the actual interest rate.

Holston, Laubach, and Williams (2017) use the Kalman filter to jointly estimate the natural rate of interest, potential output, and potential output growth rate within a multi-equation specification. Therefore, they suggest a system of equations that jointly characterizes the behavior of inflation and the output gap. In addition, the authors allow for the presence of shocks that affect the output gap and inflation but not the natural rate of interest, which they define as a low-frequency concept. Consequently, in the long run, the short-term shocks (temporary real shocks regarding government spending, asset prices, or credit conditions) fade away and the natural rate is affected predominantly by changes in productivity or demography. However, Kiley (2015) shows that those factors, notably the credit spread are relevant for the estimated relationship between output and interest rates. Put differently, the natural interest rate can vary in time due to the low-frequency shifts in the IS curve as a consequence of the significant structural changes mentioned above, and not due to the cyclical shifts in the IS curve – for instance the precautionary savings creation during recession periods (Scheerová 2017).

The Holston, Laubach, and Williams model (2017) derives the natural interest rate based on the neoclassical growth theory. It states that the equilibrium real rate is an increasing function of the trend growth rate of the output (Scheerová 2017). The reduced form of the model thus contains several equations.

The first equation i.e. the IS curve (aggregate demand equation) illustrates the output gap,  $\tilde{y}_t$ , characterized as the percentage deviation of real from potential GDP, as a function of its lags, the lags of the differential between the short-term real interest rate  $r_t$  and the natural rate of interest  $r_t^*$  and an uncorrelated error term  $\epsilon_{\tilde{y},t}$ .

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2} \sum_{j=1}^2 (r_{t-j} - r_{t-j}^*) + \epsilon_{\tilde{y},t}$$

The output gap lag structure and the error term are set to consider short-term dynamics and temporary shocks, while structural shifts in the relation between the output gap and the real interest rate (the long-term output gap dynamics) are reflected in the natural rate variations.  $a_r$  is the slope coefficient of the IS curve, which we initialized at -0.0025, following the HLW (2017) approach. The inclusion of the difference between the real and natural rates of interest indicates that the estimate of the latter has to be compatible with the real interest rate that would be obtained in an environment of stable inflation if the output gap were closed.

The second equation (i.e. the backward-looking Phillips curve) displays the inflation dynamics ( $\pi_t$ ). It is defined as a function of its own lags and lagged output gap.

$$\pi_t = b_\pi\pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y\tilde{y}_{t-1} + \epsilon_{\pi,t}$$

Factually, in this model inflation does not depend on expected inflation, which instead, we only proxy with the lags of inflation. The existence of significant stickiness in the formation of prices in the Macedonian economy is one argument in support of this assumption. This is in line with recent research based on a DSGE model with financial frictions by Copaciu et al. (2021) which concludes that the highest degree of stickiness is found for prices of domestic goods. We relax this assumption in our benchmark model and in our additional model, in which we construct a hybrid Phillips curve where current inflation depends on both past and expected inflation. In brief, the first and the second equation represent the measurement equations of the model in a state space form.

The natural rate of interest depends only on real factors, specifically the annualized trend growth rate of the output  $4g_t$  and a series of long-term random factors (associated, for example, with changes in financial deregulation, rate of time preference of households, intertemporal elasticity of substitution, population growth, global saving, institutional changes or uncertainty), which are denoted by the variable  $z_t$ . This unobserved variable follows a random walk process.

$$r_t^* = 4g_t + z_t$$

$$z_t = z_{t-1} + \epsilon_{z,t}$$

In the original HLW (2017) model, the authors suppose a one-for-one relationship between the trend growth rate of output and the natural rate of interest. Our specification corroborates this assumption as well. This relationship makes the natural interest rate inherently dependent on the dynamics of the potential GDP, and essentially on the underlying GDP series published by the official statistics that is used to recover this latent variable. Here, one peculiarity of the Macedonian data should be emphasized, which is not characteristic of the US or the euro area data, for example, and that is the unusually high volatility of the Macedonian GDP series published by the State Statistical Office. Ex-ante, one would expect that some of this volatility would pass on to the estimate of the natural rate. Unfortunately, this problem may be considered endemic in research for economies such as the Macedonian.

The final equation relates to potential output  $y_t^*$ . It is supposed to follow a random walk and time variation is allowed in its growth rate, which in turn behaves according to a random walk model. The model permits temporary shocks to both the level of potential output and its growth rate through the inclusion of the error terms  $\epsilon_{y^*,t}$  and  $\epsilon_{g,t}$ .

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_{y^*,t}$$

$$g_t = g_{t-1} + \epsilon_{g,t}$$

The latter four equations represent the transition equations of the state-space model.

In addition, to specify the lags built into the model, in our case the restrictions included in the HLW (2017) were taken as the initial reference. To this end, the output gap and inflation equations have a fairly broad lag structure. Thus, for the output gap polynomial a structure of two lags is set for the output gap itself. We decided to include two lags of the output gap in the IS

equation, in line with Laubach and Williams (2003), which through estimation concluded that such a structure is appropriate and in line with previous relevant studies. A structure of two lags is set for the real interest rate gap to take account of the transmission lags of monetary policy to output as well as for reasons of parsimony, with a similar assumption as in HLW (2017) that the lags have equal weights (however, this assumption is not rejected by the data). In the inflation equation, four lags are included to capture the stickiness in the formation of prices in the Macedonian economy, and the restriction that the coefficients sum to unity is imposed. Further, for reasons of parsimony, an additional restriction requires the coefficients of lags two to four to be equal. The decision to use a single output gap lag is consistent with HLW. Moreover, following HLW (2017) we impose the constraints that the slope  $a_r$  of the IS equation is negative and the slope  $b_y$  of the Phillips curve is positive. These authors view these as minimal priors on the structure of the model that, in the event, facilitate the convergence of the numerical optimization during estimation. Moreover, this specification seems to be sufficient to characterize the cyclical episodes in the Macedonian economy based on the proposed model (empirical validation of this statement provides Figure 5 in the Appendix -- Output gap, HP filter vs HLW model estimate).

The empirical implementation of the HLW (2017) model proceeds in three steps.

Step 1: By omitting the interest rate gap from the IS equation and by assuming that the trend growth rate is constant, we estimate a simpler model to recover a measure of potential output. To help convergence in estimation, the model coefficients were initialized by their OLS estimates (or very close approximations of them) that incorporate the HP filter estimate for the output gap. To this end, the slope coefficient in the Phillips curve, which was initialized at 0.0025 implies a rather flat Phillips curve, (or inflation which is insensitive to the output gap). However, this is empirical factuality since it closely reflects the value of the slope coefficient from the auxiliary OLS estimate of the Phillips curve. The standard error of the innovation of the IS equation is assumed to be some very small number (i.e. it is initialized at 0.05). In parallel, the standard error of the innovation in the inflation equation reflects the one obtained by the corresponding OLS estimate. We compute the exponential Wald statistic of Andrews and Ploberger (1994) for a structural break with an unknown break date from the first difference of this preliminary estimate of the potential output to obtain the median unbiased estimate of  $\lambda_g$ .

Namely, the natural interest rate and trend growth rate are likely to be subject to the pile-up problem for non-stationary processes explained by Stock (1994). In most of the samples, variations of natural interest rate and trend growth rate representing the permanent components of the time series are likely to be very small relative to the large transitory components included in the series. Because of this, the MLE estimator of the standard deviations of their changes (innovations) will be biased toward zero. The problem is tackled by median unbiased estimation of coefficient variance/standard deviation in a time-varying parameter model (Stock and Watson 1998; Scheerová 2017). The values of the Exponential Wald statistic are compared with the table provided in Stock and Watson (1998), Table 3, and converted into the median unbiased signal-to-noise ratios  $\lambda_g$  using the table. The ratio is then simply plugged into the formulas provided by Stock and Watson:

$$\lambda_g = \frac{\sigma_g}{\sigma_{y^*}} = \frac{sd(g_t)}{sd(y^*)}$$

as a relationship between the standard deviations of trend growth rate  $g_t$  and the potential output.

Step 2: The second step consists of imposing the estimated value of  $\lambda_g$  from the first step, followed by the inclusion of the real interest rate gap in the output gap equation under the assumption that  $z_t$  is constant. The real interest rate enters the model as an exogenous variable. Following HLW (2017), we construct it as the nominal interest rate net of a four-quarter moving average of past inflation. Admittedly, we use past inflation as a proxy for future (expected) inflation only as an initial assumption, which we relax in the other models that we construct. Namely, as we previously mentioned, our aim is in the first step to apply the seminal HLW model on Macedonian data, and then to continue with our benchmark model which includes forward-looking fully model-consistent expectations. This would help us understand how these different model assumptions may affect our conclusions. The slope coefficient  $a_r$  in the IS curve was initialized to -0.0025. We estimate the five model equations and apply the exponential Wald test for an intercept shift in the IS equation at an unknown date to obtain an estimate of  $\lambda_z$ .

$\lambda_z$  gives us the second restriction that needs to be imposed on the general model:

$$\lambda_z = \frac{\sigma_z a_r}{\sigma_{\tilde{y}} \sqrt{2}} = \frac{sd(z_t)}{sd(\tilde{y})}$$

as a relationship between the standard deviations of  $z_t$  and the output gap. The equation differs from the one related to  $\lambda_g$  due to the following reasons. The term  $\sqrt{2}$  is used since in the IS equation of the general model it is assumed that the output gap is influenced by two lags of the interest rate gap and the current interest rate gap is determined by the current  $z_t$ , as mentioned in the description of the variables in the general model. Such specification implies that the potential output is affected by  $z_t(-1)$  and  $z_t(-2)$  via the coefficient  $a_r$ .

Step 3: In the final stage of the estimation, the above-mentioned restrictions on variances are imposed on the whole system of all equations in the general model, and the Maximum Likelihood method is used to estimate its parameters. The estimated values of the slope parameters  $a_r$  and  $b_y$  remain close to -0.0025 and 0.0025, respectively, (i.e. they are only marginally lower than the above-mentioned initial values in absolute terms) and are not significant at any conventional significance level. The lack of significance in IS and Phillips curve slopes is one of the reasons why the estimates of the natural interest rate and the potential output are highly imprecise and barely identified. However, the precision of the HLW (2017) estimates dramatically falls when either the output gap is insensitive to the real interest rate gap (flat IS curve), or inflation is insensitive to the output gap (flat Phillips curve). In those cases, it is not possible to uniquely identify the unobserved growth and non-growth components of  $r^*$  from the data. Unfortunately, those two cases are empirically relevant according to a wide set of estimates reported in the existing literature (Fiorentini et al. 2018). Moreover, the variability of the natural interest rate components (trend growth rate and other factors) allowed in the model, or the value of initial parameters can also significantly alter the resulting estimates (Scheerová 2017). The associated uncertainty could prevent the straightforward use of the estimated natural interest rate in policy applications. As a result, policy decisions, should not take the HLW (2017) type of estimate of the natural interest rate for the Macedonian economy into much consideration. In other words, this estimate should be taken as indicative, and not as an incentive for a change in monetary policy.

Against this background, the natural rate of interest is a theoretically reasonable notion for assessing the monetary policy stance. Deriving the natural rate is another question, however. Given that it is an unobserved variable, it has to be extracted from data by imposing some identifying assumptions. To this end, the

actual estimate is almost fully based on the assumptions decided by the researcher and imposed on the data going into the economic model. In other words, there are a number of issues to be addressed by the estimation process. Estimation concerns are likely not induced by omitted variables exclusively, and the omitted equations might be an issue as well. Consequently, the estimated trajectory of the unobserved variables like equilibrium interest rate or trend growth rate can reflect movements of other macroeconomic variables influencing the economy, which were not embedded in the model, so the relations become spurious (Scheerová 2017).

In addition, Laubach and Williams model, but also many others, focusing on long-term developments are inherently prone to potential imprecision and poor reliability notably due to the presence of a large number of unobserved variables in them (Scheerová 2017).

Another potential source of the low robustness is a one-sided filtering technique for real-time estimates, in which the estimate at a defined point in time is based uniquely on the information available at that time. As a consequence, the one-sided estimates normally include much more noise (Scheerová 2017).

We believe those are the main limitations that one should take note of.

Against this background, policy inference did not take the time-varying estimates of the equilibrium interest rate into much consideration.

All this said, the HLW model serves as a starting point in our analysis since it is a seminal model used for estimating the natural rate of interest for many countries, the USA and the euro area included. In addition, it is considered simple, tractable, and rather easy to implement on a regular and more frequent basis. However, our main focus is put on the benchmark MAKPAM model that is used for comprehensive policy purposes. Namely, this core model addresses most of the disadvantages of the HLW model, most prominently, it has a much richer model structure able to capture more consistently the long-term factors affecting the natural interest rate, it incorporates forward-looking model-consistent expectations and is closed by a policy rule. For that reason, we analyze the results from the HLW model only in comparison with the benchmark MAKPAM results, to gain a broader understanding of the concept of natural interest rate, which as we stated earlier is a very elusive concept, sensitive to the chosen estimation methodology. In other words, we opt to be very prudent in our approach, to obtain the most robust results possible in order to understand how our conclusions might

change. This commitment of ours is shown also by estimating a third model with forward-looking inflation expectations, but which is different in its structure from the other ones in that it considers another determinant of the natural rate of interest, which some authors argue that is relevant for small open economies, namely the real equilibrium exchange rate.

In what follows, we present the core and the additional small-scale model for the estimation of the natural interest rate and discuss the results.

#### **4.2. A structural model of the Macedonian economy – modified MAKPAM model**

As a benchmark, we utilize the Macedonian Policy Analysis Model<sup>2</sup> (MAKPAM) in order to quantify the natural rate of interest for the purpose of this research, in which the main modification that is made pertains precisely to the natural rate equation<sup>3</sup>. MAKPAM model is a structural, New Keynesian gap model, which can capture the broad macroeconomic relationships in the Macedonian economy, as a small open economy with a fixed exchange rate regime. It is a linear, calibrated model, in which all variables are in natural logarithms, expressed as percentage deviations from the trends. The trends (equilibria) and the deviations (gaps) are unobserved and the Kalman filtration is used for their identification. The model has three main building blocks: the real economy block, the price block, and the monetary policy and balance of payments block. A detailed explanation of the model is out of the scope of this paper. Instead, we focus only on the key model equations, determining the natural rate of interest. The main difference with the previously used identification methodology of HLW (2017) is that this model represents a structural economic model, which is important given that the whole model structure, not only the model equations that are used to identify the natural rate of interest, are being utilized to appropriately determine the rate. MAKPAM is also an open economy model, contrary to the HLW (2017) model. The model is also closed by an exchange regime-specific monetary policy rule and incorporates forward-looking model-consistent expectations, as opposed to the absence of policy rule and the moving average of past inflation used in the HLW (2017) model. In addition, calibration instead of Bayesian estimation is used to find the model parameters that enable the model to best fit the data, which is verified by both data filtration, impulse responses, and in-sample simulations. In what follows, the main model equations are presented.



According to the Fisher equation, the real interest rate ( $R$ ) is the difference between the nominal interest rate ( $I$ ) and model consistent expected inflation ( $\pi_{exp}$ ):

$$R_t = I_t - \pi_{exp}$$

The real interest rate is then decomposed into the natural rate and the gap. The interest rate gap is defined as:

$$R_{gap,t} = R_t - R_{eq,t}$$

It is a function of the real interest rate  $R_t$  and the natural interest rate  $R_{eq,t}$ .

In line with Laubach and Williams (2003) and Holston, Laubach, and Williams (2017), we assume that the natural rate of interest is determined by the annual growth rate of the potential GDP  $\Delta 4Y_{eq,t}$  and a time-varying unobserved component  $z_t$ :

$$R_{eq,t} = \delta R_{eq,t-1} + (1 - \delta)\Delta 4Y_{eq,t} + z_t$$

However, it should be noted that unlike LW (2003) and HLW (2017), we have also introduced some persistence in the natural rate equation.

Everything that affects potential growth, such as investment, labor supply, or government shocks, will proportionately affect the natural rate of interest. In the model, the growth of potential GDP is a weighted sum of the equilibrium growth rates of private consumption, government consumption, investment, exports, and imports, which in turn are modeled as an AR(1) process of the following functional form:

$$\Delta C_t^{eq} = \vartheta \Delta C_{t-1}^{eq} + (1 - \vartheta)\Delta C_{ss}^{eq} + \varepsilon_t^{\Delta Ceq}$$

The natural rate is also assumed to be affected by transitory shocks through the auto-regressive process  $z_t$ , which represents all the other determinants mentioned before which are not directly linked to domestic potential growth:

$$z_t = \mu z_{t-1} + \varepsilon_t^z$$

As a result, the natural interest rate should fluctuate over time with changes in its long-term fundamentals.

It should be pointed out that any equation in the model that includes either the natural interest rate or

the real rate is used as a cross-restriction for its identification (Hlédik and Vlček 2018). The first such restriction comes from the real economy block (output gap). However, the output gap is highly disaggregated in the model, meaning that it is a weighted sum of the gaps of private consumption, government consumption, investment, exports, and imports. Thus, the equations directly relevant for the natural interest rate identification are only the ones for the private consumption gap and investment gap:

$$C_{gap,t} = \alpha_0 C_{gap,t-1} + \alpha_1 R_{gap,t-1} + \alpha_2 RDI_{t-1} + \varepsilon_t^C$$

The private consumption gap  $C_{gap,t}$  is a function of its lag, lagged real interest rate gap  $R_{gap,t-1}$  and lagged real disposable income gap  $RDI_{t-1}$ , where the latter is a sum of the real wage bill, real pensions, and real private transfers gaps, all modeled with separate behavioral equations.

$$J_{gap,t} = \beta_0 J_{gap,t-1} + \beta_1 R_{gap,t-1} + \beta_2 X_{gap,t-1} + \beta_3 FDI_{gap,t-1} + \beta_4 YF_{gap,t-1} + \varepsilon_t^J$$

The real investment gap  $J_{gap,t}$  is a function of its lag, lagged real interest rate gap, lagged exports gap  $X_{gap,t-1}$ , lagged FDI gap  $FDI_{gap,t-1}$  and lagged foreign effective demand gap  $YF_{gap,t-1}$ .

Another restriction is the equation for the private transfers, as a specific factor for the Macedonian economy.

$$PT_{gap,t} = \gamma_0 PT_{gap,t-1} + \gamma_1 R_{gap,t-1} + \gamma_2 DD_{gap,t-1} + \gamma_3 (\pi_{\Delta 4,t-1} - \pi_{ss}) + \varepsilon_t^{PT}$$

The real private transfers gap  $PT_{gap,t}$  is a function of its lag, lagged real interest rate gap, lagged domestic demand gap  $DD_{gap,t-1}$  and the deviation of the inflation from the steady state  $(\pi_{\Delta 4,t-1} - \pi_{ss})$ . This equation also enters into the monetary policy rule, which differs from the usual Taylor-type policy rule (in which the natural rate enters directly). Instead of the Taylor rule, we use an arbitrage condition stating that domestic and foreign interest rates can differ only if the risk premium is different from zero. In other words, the policy rate is a function only of the foreign interest rate  $i_t^*$  and the risk premium  $prem_t$ , and the latter depends, among the other, on the private transfers as one of the components (together with net export and FDI) that determine the foreign reserves flow gap  $flow_{gap,t}$ . Thus, this equation is especially important, since it allows the monetary policy rule to serve as an indirect restriction for the natural rate of interest.

$$\begin{aligned}
 i_t &= i_t^* + prem_t + \varepsilon_t^i \\
 prem_t &= -\tau_1(flow\_gap_t + flow\_gap_{t+1} \\
 &\quad + flow\_gap_{t+2} + flow\_gap_{t+3})/4 \\
 flow_{gap,t} &= f(NX_{gap,t}, FDI_{gap,t}, PT_{gap,t})
 \end{aligned}$$

The price block consists of an aggregated New Keynesian Phillips curve (without the administered prices component):

$$\begin{aligned}
 \pi_t &= \omega_1\pi_{t+1} + (1 - \omega_1)\pi_{t-1} \\
 &+ (1 - MC_{SS})(\omega_2 Y_{gap} + \omega_3 MC_{SS} Q_{gap,t}) + \varepsilon_t^\pi
 \end{aligned}$$

According to this equation, consumer price inflation  $\pi_t$  is a linear combination of one-quarter ahead inflation, one-quarter lagged inflation, the output gap  $Y_{gap}$  and the real exchange rate gap  $Q_{gap,t}$ . Total inflation is then simply obtained by adding the administered prices component. The output gap captures the domestic price pressures, whereas the real exchange rate captures the import price pressures. Its depreciation (appreciation) translates into higher (lower) domestic inflation, due to the rise (decline) of the price of imported factors of production.

The exchange rate channel is included to capture the fact that the Macedonian economy is a small and open economy. The real exchange rate  $Q_t$  is defined as:

$$Q_t = e_t - p_t + p_t^f$$

where  $e_t$  is the nominal exchange rate,  $p_t$  is the domestic price level, and  $p_t^f$  is the foreign price level. The real exchange rate gap then is:

$$Q_{gap,t} = Q_t - Q_{eq,t}$$

Besides the import price pressures, it also captures the effects on the output gap from its impact on exports, which is a standard transmission mechanism in small and open economies.  $Q_{eq,t}$  is the equilibrium price level, governed as:

$$\Delta Q_t^{eq} = \varphi \Delta Q_{t-1}^{eq} + (1 - \varphi) \Delta Q_{SS}^{eq} + \varepsilon_t^{\Delta Q_{eq}}$$

### 4.3. Empirical results

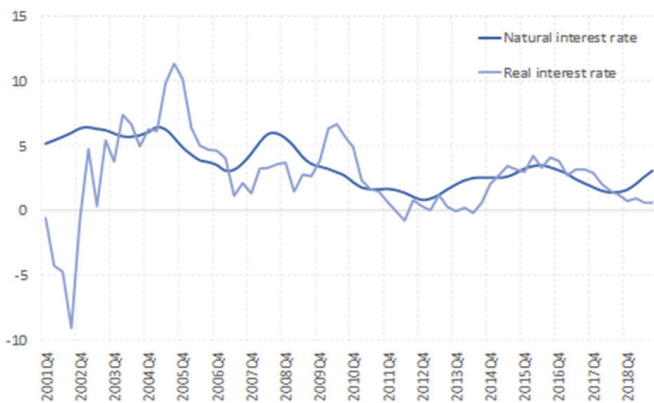
This section first discusses the results for the natural rate of interest, as identified with the benchmark MAK-PAM model, and then compares them with the estimates from the Holston, Laubach, and Williams (2017) model. In addition, within the section we focus on the

MAKPAM results in order to analyze the monetary policy stance as well as to compare the Macedonian natural interest rate with the one of the euro area.

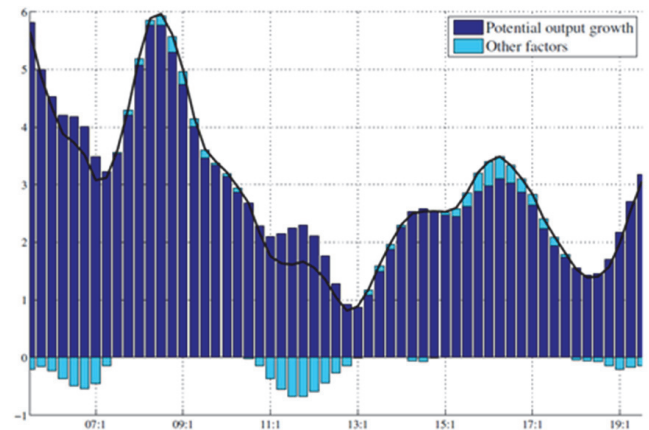
#### 4.3.1. Estimates of the natural rate of interest

The results from the modified MAK-PAM model are reasonable, suggesting that the natural rate of interest moves in tandem with the real interest rate, both of which follow downward dynamics. They suggest that on average, the level of the natural rate of interest was 3.5% in the period 2002Q1-2019Q3, slightly above the average real interest rate. However, two different phases can be detected in the dynamics of the natural rate in the analyzed period (Figure 3). First, prior to the global economic crisis, the natural rate was higher, hovering around 5%. During this period, the Macedonian economy experienced high growth of potential GDP, which thus had the highest contribution to the rate (Figure 4). However, since the onset of the global crisis, the natural rate of interest started declining (from 2008Q4 onwards), driven mainly by the deceleration of the potential GDP growth. In the after-crisis period, the natural rate of interest nearly halved, amounting to 2.4%, which is lower than the historical average by around 1 percentage point. However, there are positive signs that the natural rate of interest is again on the way up at the end of the estimation period, which leaves space for the monetary policy to efficiently create a negative real interest rate gap to stimulate the economy if needed, even in the case of low or slightly negative inflation. For the overall period, we find that potential growth has a downward sloping trend, which explains the decline in the natural rate of interest, while the component  $z$  is more stable (Figure 1 in the Appendix). It is also evident that the natural rate displays certain volatility, an observation that is again consistent with the high variability of potential growth of Macedonian GDP. However, it should be pointed out that this volatility decreased in the period after the global crisis (by around 15%), in line with the reduction of the volatility of potential growth. Here, one peculiarity of Macedonian data should be noted, and that is the unusually high volatility of the Macedonian GDP series published by the official statistics, which in turn translates into volatile estimates of the potential GDP and hence the natural interest rate, as previously mentioned. Similarly to Brzoza-Brzezina (2003), we also find evidence that the natural rate is, in general, a pro-cyclical variable – with a few exceptions (such as the last period) its level is increasing in expansionary times and decreases in recessionary times (Figure 2 in the Appendix).

**Figure 3. Natural interest rate and real interest rate from MAKPAM, in %**



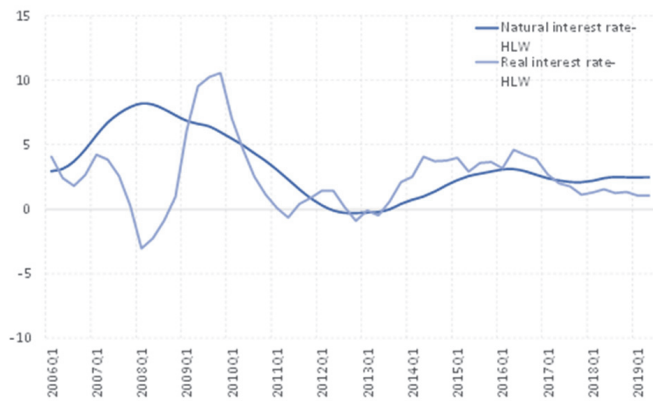
**Figure 4. Natural interest rate and decomposition (MAKPAM)**



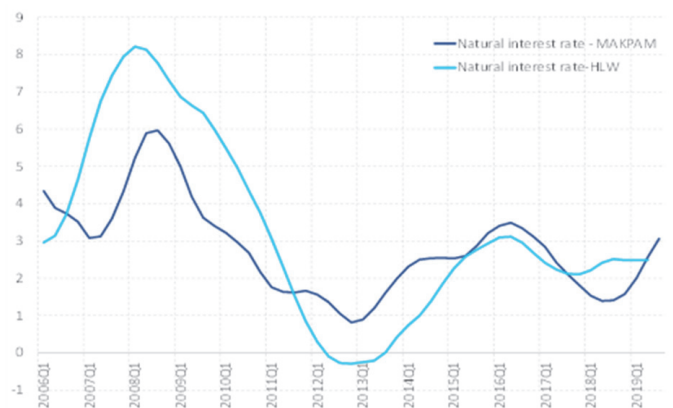
The comparison with the estimates from the HLW (2017) model (Figure 6) shows that the natural rate of interest follows very similar dynamics in the two models and has a similar cyclical nature. Similarly to the MAKPAM estimates, the HLW natural rate of interest estimate peaked prior to the global financial crisis, i.e. in 2007-2008, and turned sharply lower afterward. In fact, the post-crisis averages of the natural rate are identical according to both the MAKPAM and the HLW (2017) model (2.4%). However, analyzed on average for the 2006-2019 period, the HLW estimate is higher by 0.5 percentage points than the MAKPAM estimate (3.3% according to the HLW model and 2.8% according to the MAKPAM model). Moreover, three different periods can be observed in which the two estimates differ. First, up until the second half of 2011 (prior to

the European debt crisis and the global economic crisis) the level of the natural rate of interest based on the HLW (2017) methodology tends to be above the MAKPAM estimate in a systematic way. Second, the HLW model points to a lower natural rate of interest for the 2011Q3-2017Q2 period. During the European debt crisis, the HLW estimate of the natural interest rate even enters into negative territory (from mid-2012 until mid-2013). Third, the HLW estimate is again somewhat above the MAKPAM estimate from 2017Q3 onwards. Another difference is that the natural rate of interest estimated from the HLW model is more volatile compared to the MAKPAM estimate.

**Figure 5. Natural interest rate and real interest rate from HLW (2017), in %**



**Figure 6. Comparison of the two estimates**



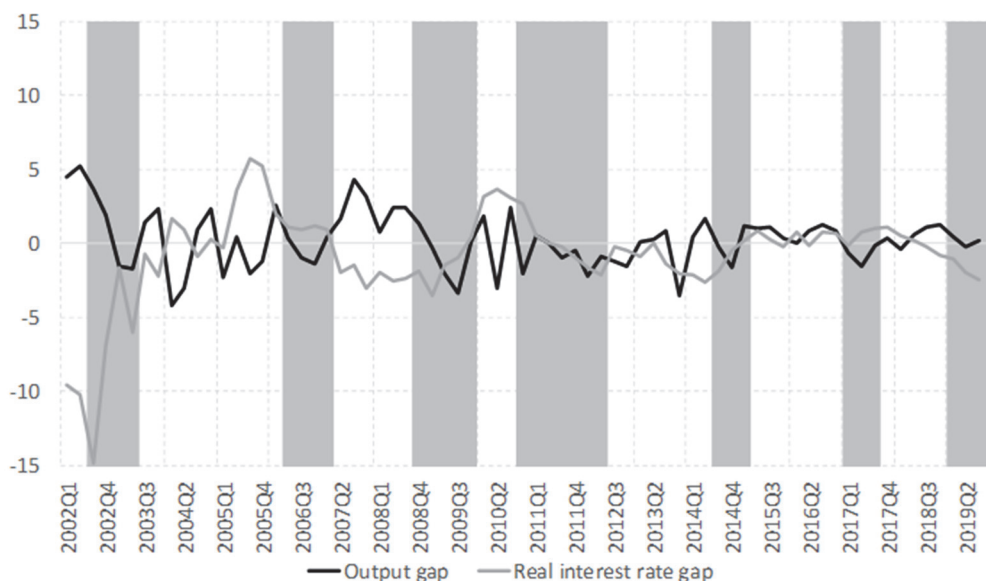
### 4.3.2. Assessment of the monetary policy stance and comparison of the Macedonian and the euro area interest rate using the MAKPAM model

In the following subsection, we proceed by analyzing the monetary policy stance using the benchmark MAKPAM model estimates and we provide a comparison with the euro area natural interest rate.

It is well known that our focus should not be only on the level of the natural rate of interest per se, but also on the difference between the real rate and the natural rate (the gap) since the latter is indicative of the stance of monetary policy and its implications for the domestic economy. For example, if the central bank intends to stimulate the economy, not only it should lower the interest rate, but it should also bring it below the natural rate of interest and produce a negative real interest rate gap. This will allow the actual output to grow at a faster rate than the potential output so that eventually the discrepancy between the two would be closed. It can be seen from Figure 7 that during the analyzed period, the gap between the real interest rate and the natural rate had divergent effects on the domestic economy and that in general, when the real rate gap was negative, the output gap tended to increase, and vice versa. Namely, from 2004 until the beginning of 2007, when the Macedonian economy experienced high GDP growth, the real interest rate gap was positive, so the real interest rate did not contribute to further fueling this growth. Conversely, the results suggest that in the period just before and during the global economic crisis as well as in the

period during the Greek debt crisis, the real interest rate was supportive of the real economy. On the other hand, in 2010-2011, we again observe a tightening of the monetary policy conditions through the real interest rate. From 2016 to 2018, the real interest rate gap was relatively small, and the real interest rate was hovering near the natural rate. However, we observe that at the end of our sample, the level of the real interest rate falls again below the natural rate, thus pointing to accommodative monetary policy conditions in the recent period. These conclusions broadly hold also according to the HLW (2017) model results. This accommodative stance of the monetary policy has been reasonable, given that according to the estimated data for GDP in the first three quarters of 2019, the Macedonian economy was in the recessionary phase of the cycle, and there were no inflationary pressures from the demand side (Figure 7)<sup>2</sup>. It should be noted that for the purpose of identifying whether the economy is in an expansionary or recessionary phase of the business cycle in this paper, we use the BBQ algorithm for detection of cyclical turning points, developed by Harding and Pagan (2002). A detailed explanation of this method is out of the scope of the paper and can be found in Miteski and Georgievska (2016) instead. On the other hand, if we analyze the movements of the output gap in this period as a more simplistic way of looking at the cycle, it can be seen in Figure 7 that it gravitates around zero in the recent sample period, indicating no substantial expansions or contractions of the economy.

**Figure 7. Output gap, real interest rate gap, and the phase of the economy (MAKPAM)**



\*Shaded areas depict recessionary phases, determined with the turning point methodology of Harding and Pagan (2002).

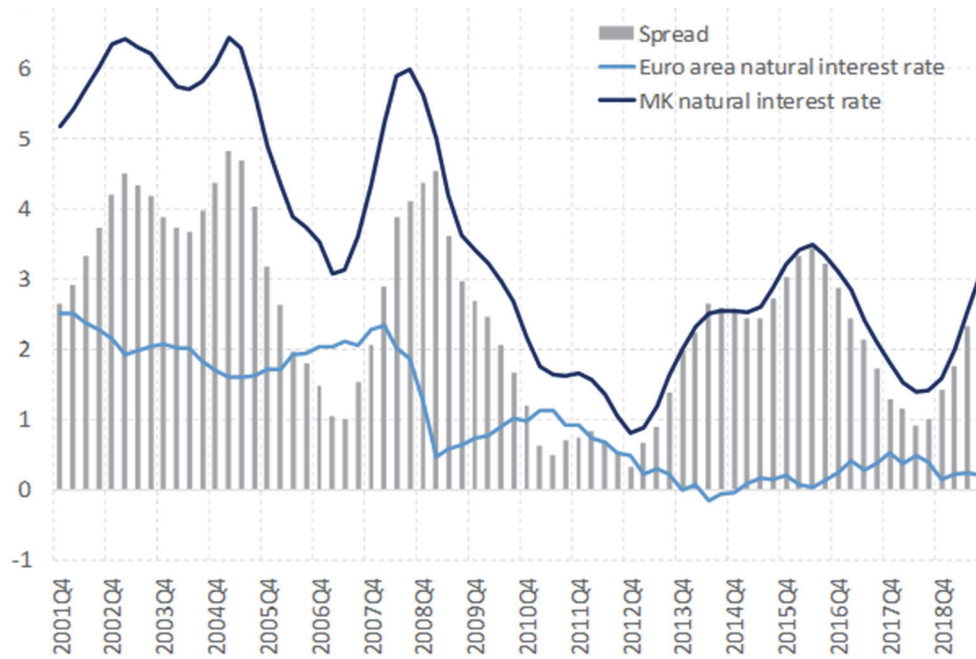
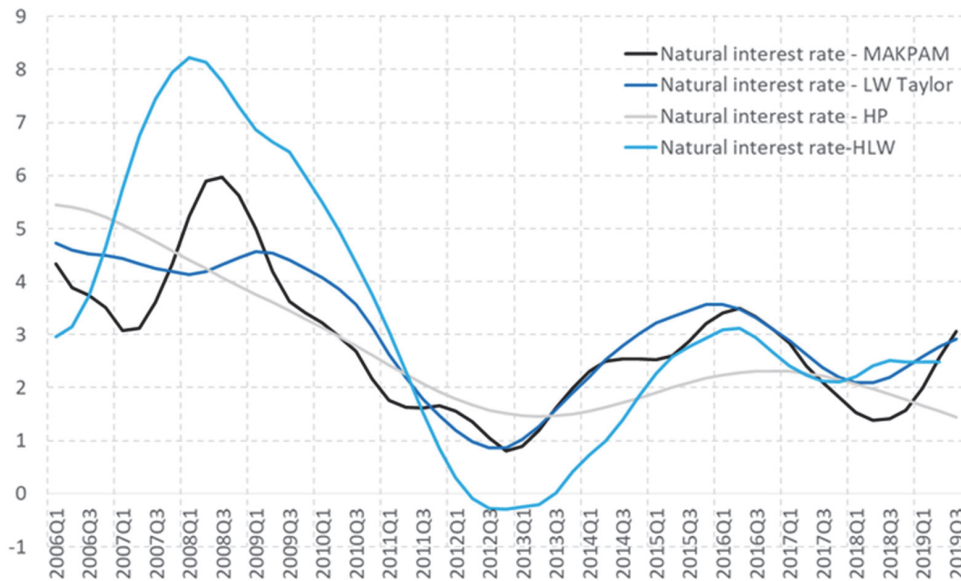
**Figure 8. Comparison of Macedonian and euro area natural rate of interest**

Figure 8 shows a comparison between the Macedonian and the euro area natural rates of interest. It can be seen that the Macedonian natural rate of interest follows a similar downward trend as the euro area natural rate of interest<sup>3</sup>, although the level is expectedly higher. The average difference between the two natural rates of interest is 2.5 percentage points<sup>4</sup> for the whole period. Although, it is evident that the differential has narrowed in the period after the global economic crisis (by around 1 percentage point) as compared to the before-crisis period<sup>5</sup>, which seems to be consistent with the narrowed potential growth differential between the two countries. The similar movements of the euro area and Macedonian natural interest rates imply that developments in the euro area, which are modeled through the inclusion of the foreign effective demand indicator<sup>6</sup>, may also play a role in the dynamics of the Macedonian natural rate of interest, besides the domestic factors, although the latter appears to have the predominant role.

## 5. Additional specifications

In this section, we provide a comparison of the previous estimates of the natural rate with the ones from some alternative specifications<sup>7</sup>.

Hlédik and Vlček (2018) argue that the natural rate of interest in a small open economy that follows a path of convergence to the more advanced economies is a function of the potential GDP growth adjusted for the appreciation of the real equilibrium exchange rate. We follow their logic and develop a similar model for the Macedonian economy with forward-looking inflation expectations, which is also closed by a Taylor-type monetary policy rule, unlike the MAKPAM model. As can be seen from Figure 9, these results are similar to the MAKPAM and HLW estimates, especially in the period after the European debt crisis. Additionally, we compare the estimates from the models with the results from the univariate HP filter. In this case, the natural interest rate identified with the economic models deviates from the HP estimate and the difference is more pronounced at the end of the sample (the well-known end-point bias). Namely, we observe that the HP estimate is biased in the direction of the recent movements of the actual interest rate, which is not the case with the economic models. However, the HP estimates are not seen to be very in line with the theoretical concept of natural interest rate and should serve only as an indication.

**Figure 9. Comparison of natural interest rates obtained by different models**

## 6. Conclusion

The main goal of this paper was to quantitatively estimate the level of the natural rate of interest in the Macedonian economy, which is a significant yet underexplored issue. This issue is important since it allows central banks to assess their monetary policy stance at any given time and hence make appropriate monetary policy decisions. Yet, any attempt to model the natural rate of interest is challenging, especially in an economy such as the Macedonian, due to the highly volatile macroeconomic series. To the best of our knowledge, this paper is the first to estimate the natural rate for the Macedonian economy. To this end, the estimation was made using different models, such as the Holston, Laubach, and Williams (2017) model and the full-fledged country-specific structural MAK-PAM model. We identify the natural rate of interest by assuming that its dynamics depends on the movement of the potential GDP growth over time, as well as on other determinants generally captured by a random process. In the Holston, Laubach, and Williams models that we develop, we use two equations (IS curve and Phillips curve) to identify the natural rate of interest. In an alternative specification we include the real equilibrium exchange rate as a determinant of the natural interest rate and we add a Taylor-type monetary policy rule, among the other changes, whereas within the MAK-PAM model the entire model structure, not only the several cross-restriction equations were used to estimate this unobserved variable, simultaneously with many other variables. Despite the substantial differences in model specification, the key finding

from the models is in essence very similar and indicate that the natural rate of interest in the Macedonian economy has declined over time. Namely, it is evident from the results that the natural rate of interest in the recent period fluctuates around a new, lower level than before the global crisis. This finding is in line with other recent international empirical studies (see for example, Rachel and Smith 2015; Holston, Laubach, and Williams 2017), which also find evidence of a large decline in the natural rate of interest since the onset of the global economic crisis. In addition, the results show that the natural rate of interest follows very similar dynamics in all of the models and has a similar cyclical nature. The main difference in the results is that, on average, the level of the natural rate of interest based on the HLW (2017) methodology is somewhat higher and more volatile than the MAK-PAM estimates. However, both results point to accommodative monetary policy conditions in the Macedonian economy in the recent period. The decomposition using the MAK-PAM model shows that over the estimated period, the dynamics of the natural rate was predominantly determined by the decline of the Macedonian potential output growth, with other factors, such as household preferences, also being at work but without a major role. From the aspect of robustness, the results of the other alternative models are also very comparable and confirm the main findings of the study. Overall, these findings provide useful insights into the natural rate of interest dynamics, which can provide guidance to Macedonian policymakers about the appropriate changes in monetary policy implementation.

## Endnotes

- 1 Since January 2002 the National Bank has been implementing monetary strategy of fixed nominal exchange rate of the Denar against the Euro.
- 2 This figure also shows that the calibration of the model satisfies the cyclical profile of the Macedonian economy, i.e. that recessions are set out fairly accurately (also separately shown in Figure 3 in the Appendix). The path of other key unobservable variables that are jointly determined seems to be reasonable as well (not presented here for the sake of brevity).
- 3 The estimated natural interest rates for the euro area can be downloaded from the website of Federal Reserve Bank of New York: <https://www.newyorkfed.org/research/policy/rstar>. It also includes interest rates for USA, Canada and the United Kingdom.
- 4 The average difference between the HLW estimates of the Macedonian natural rate and the euro area natural rate is also 2.5%.
- 5 The spread before the crisis was 3.3 p.p., whereas after the crisis it was 1.9 p.p.
- 6 Foreign effective demand is calculated as the weighted sum of GDPs of the most important trading partners for the Macedonian economy. 70% of the countries included are members of the euro area.
- 7 For the sake of brevity, we do not present these specifications in the paper.
- 8 A detailed description of an earlier version of the model can be found in Hlédik et al. (2016).
- 9 In the original MAKPAM model the equilibrium interest rate is modeled as an exogenous AR process.

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APPENDIX

Figure 1. Natural rate of interest and potential growth (left panel), and component z (right panel) estimated with the MAKPAM model

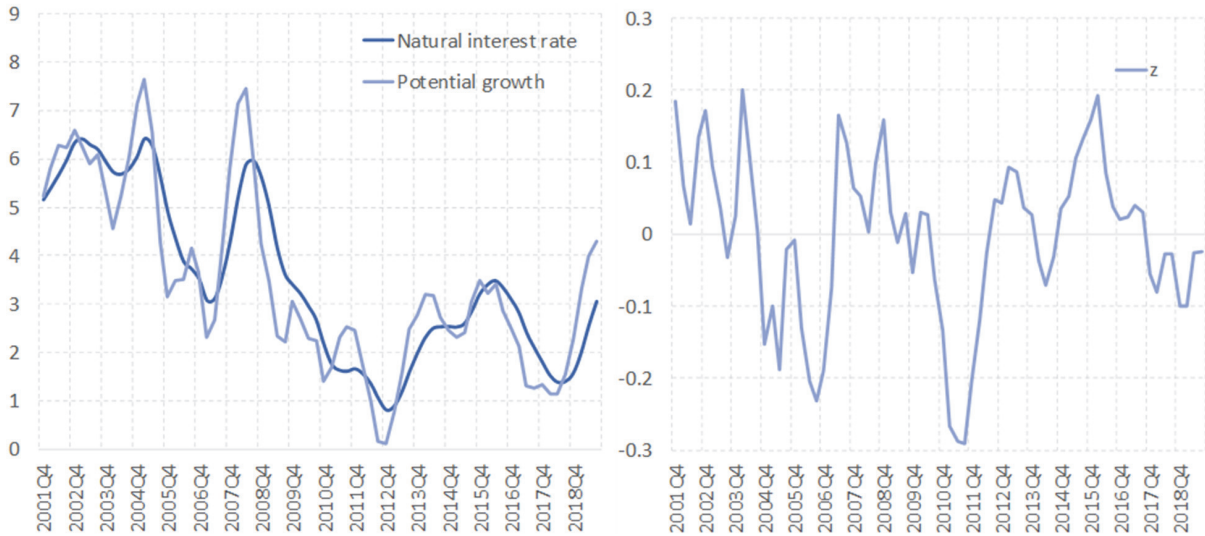
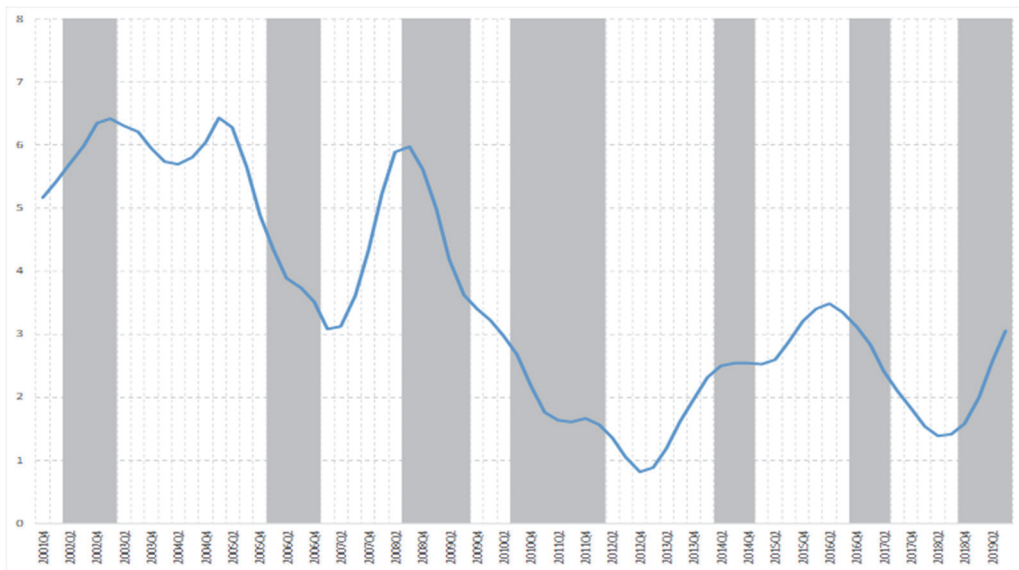


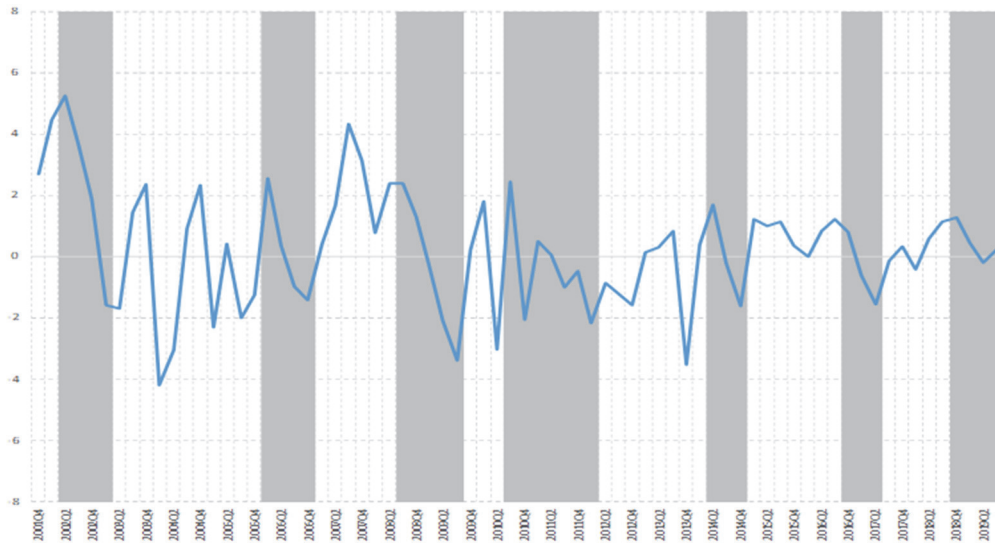
Figure 2. Estimate of the natural rate of interest using the MAKPAM model



\*Recessionary phases are determined with the turning point methodology of Harding and Pagan (2002).

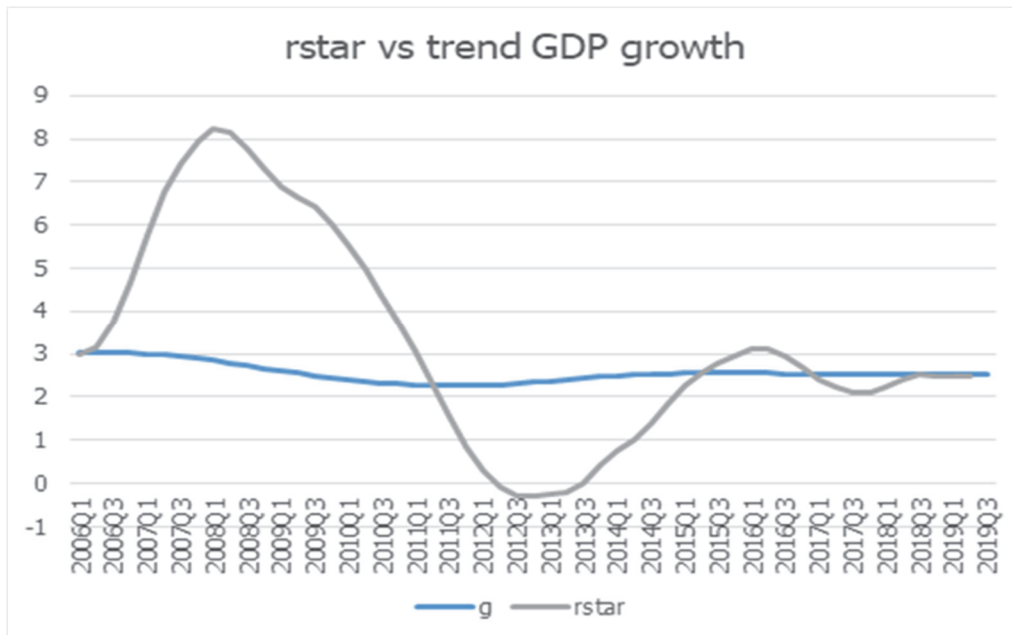


**Figure 3. Estimate of output gap using the MAKPAM model**



\*Recessionary phases are determined with the turning point methodology of Harding and Pagan (2002).

**Figure 4. Natural rate of interest and potential output growth estimated with the HLW (2017) model**



**Figure 5. Estimates of output gap using the HLW (2017) model**