

PERFORMANCE GAUGING OF PORTFOLIO: LUENBERGER DISTANCE FUNCTION APPROACH ON SARAJEVO STOCK EXCHANGE

Tihana Škrinjaric

Abstract

Efficiency measurement of economic phenomena is of great importance to economists. Known terms and concepts from microeconomic theory and Data Envelopment Analysis regarding efficiency have been developed and adapted to cater to different questions and areas of economics. This paper focuses on applications in the area of finance, more specifically, portfolio optimization and Markowitz model. The paper has two goals. First is to give a concise overview of the theoretical and empirical research regarding Luenberger shortage (distance) function in portfolio optimization. The second goal is to empirically evaluate the efficiency of a portfolio on the Sarajevo Stock Exchange (SSE). The contribution of the paper is the first comprehensive evaluation of sources of inefficiencies of a portfolio which is not positioned on the efficient frontier on the SSE, as well as the Balkan region. The results of the analysis show that efficiency can be obtained with rebalancing the initial portfolio with the inclusion of transaction costs as well.

Keywords: Markowitz model, efficient frontier, measuring efficiency, distance function, Luenberger's shortage function.

JEL classification: C14, C61, G11

1. INTRODUCTION

Ever since the seminal work of Markowitz (1952, 1959) and defining the main concepts of Modern Portfolio Theory (MPT), academics and investors have been searching for the best diversified and efficient investment portfolios. This is based upon ever growing needs and new concepts (not only in MPT, but in the Post Modern Portfolio Theory as well). Measuring efficiency in economics, in general, is not a new concept. Many models in Data Envelopment Analysis (DEA) have been developed in order to measure the efficiency of different decision-making units (DMUs). The models were tailored with respect to the characteristics of DMUs and areas of application. This is especially true for the production theory in microeconomics.

One of the most famous shortage, i.e. distance function is the Luenberger (1992, 1995) one. This function measures the distance of an inefficient unit (production plan) from the efficient frontier (consisting of technologically available production plans) with

Tihana Škrinjaric, PhD
 Postdoctoral researcher
 Department of Mathematics
 Faculty of Economics and Business
 University of Zagreb
 E-mail: tskrinjar@net.efzg.hr

a given direction. Chambers, Chung and Färe (1996) introduced the directional distance function which measured technical efficiency based upon simultaneously reducing inputs and increasing outputs of production. In that way, distance function represents an indicator of inefficiency. Briec et al. (2004) was the first theoretical study which combined MPT and the Markowitz model with the theory of distance functions. The basic idea is to evaluate the inefficiency of a given portfolio relative to the efficient frontier which is a result of optimizing the Markowitz model. In that way, investors can apply this methodology to evaluate inefficiency and its sources of given portfolios. Moreover, it can be used in forecasting as well; when investors forecast future risks and returns and then apply the distance function of forecasted portfolios in order to improve the performance of portfolios.

By observing the existing literature on investment and portfolio analysis in Bosnia and Herzegovina, as well as the Balkan region as well, there does not exist a study which applies the aforementioned methodology in order to evaluate the (in)efficiency of investment portfolios. Thus, this research aims to fulfil this gap in the literature. Moreover, the second contribution lies upon giving a concise overview of the theoretical and empirical research in this area, which is also not found in the existing literature. The distance function methodology can be applied to any type of portfolio: individually or institutionally owned ones. Due to the availability of data, this research will undertake empirical application on the market index SASE10 on the Sarajevo market. Specific portfolio structures of any investor are not available to the author. Thus, the focus is going to be answering questions on sources of inefficiencies of a hypothetical portfolio if the investor held the SASE10 composition. The rest of the paper is as follows. The second section deals with previous related research, focusing on theory and applications as well. The third section depicts the methodology used in the empirical part of the paper, the fourth section. The final, fifth section concludes the paper.

2. PREVIOUS RESEARCH

Performance measurement in portfolio analysis has been developing ever since the Markowitz portfolio has been developed. Many of the measures incorporate different valuations and definitions of risk and return, especially in the Post Modern Portfolio Theory (see Chen 2016). Some of the most popular include: Sharpe (1964) ratio, modified Sharpe ratio (Pezier and White, 2006), risk-adjusted performance (Modigliani and Modigliani, 1997), Treynor (1961,

1962) ratio, information ratio (Sharpe 1994), Sortino ratio (Sortino and van der Meer, 1991), gain-loss ratio (Bernardo and Leodit, 2000), Sterling's ratio (Kestner 1996, Bacon 2012), Burke's ratio (Burke, 1994), Jensen's alpha (1968), etc. Many other measures can be found in Chen (2016) or Agarwal and Naik (2004) or Fabozzi and Pachamanova (2016). However, these measures observe risk and reward ratios in the majority of cases. Efficiency measurement used in this study is observed in terms of the definitions within DEA.

Luenberger (1992) introduced the notion of benefit function, which was expanded in Chambers, Chung and Färe (1996) to the directional distance function in production theory in order to measure the technical efficiency of a production unit. In this setting, the authors looked at simultaneously reducing inputs while increasing the outputs. Within the DEA methodology, this can be evaluated via the additive model (see Cooper, Seiford and Tone, 2006 for examples). The arrival to the efficient frontier within this methodology depends upon choosing the directional vector. Chambers Chung and Färe (1996), as well as Briec, Kerstens and Lesourd (2004) suggest using the coordinates of the point (unit) which is being evaluated (with a negative value for inputs due to their reduction).

In terms of incorporating this methodology within portfolio selection, the work of Briec, Kerstens and Lesourd (2004) was the first one to merge both methodologies together. In this seminal paper, authors define necessary terms (and concepts) in order to introduce distance functions which evaluate inefficiency of portfolios compared to the efficient frontier within the Markowitz portfolio framework. Authors defined the efficiency improvement possibility (EIP) function which observed simultaneously reducing portfolio risk and increasing portfolio return, as the directional distance function in mentioned Chambers, Chung and Färe (1996). The paper from 2004 is mostly theoretical, with a small empirical example at the end. Authors have evaluated the efficiency of 26 investment funds on data retrieved from Morey and Morey (1999) sample. Basic interpretations were given on different sources of inefficiencies. Since the paper from 2004 observed the Markowitz portfolio problem, it included only the first two moments of the portfolio return distribution. Briec, Kerstens and Jokung (2007) extend the analysis to three-moment portfolio, by including skewness as well. Since it is known from investment utility theory that investors seek to maximize the odd moments of portfolio return distributions and minimize the even moments, authors incorporate simultaneously maximizing the portfolio return and skewness whilst minimizing the portfolio risk. Moreover, a dual approach of optimization can be found in this

research, which allows for measuring convexity efficiency component besides the usual portfolio and allocative efficiency improvements. Again, as in previous papers, the empirical example is small at the end. The four-moment case with portfolio kurtosis was observed in Jurczenko, Maillet and Merlin (2008). This is an extensive study on over 70 pages where the complexity of analytically deriving the four-moment case can be seen. The empirical analysis in the mentioned paper is extensive as well (a European stock sample – 162 stocks, time period: June 2001 until June 2006). In order to justify the usage of higher moments, authors test for non-normality of return distributions, which was confirmed for the observed data. Moreover, since higher moments portfolio optimization is based upon utility functions, authors compare results for several different utility function forms. The results of the analysis confirmed theoretical research from the 1950s when authors concluded that the mean-variance utility functions can accurately predict the optimal portfolio structure with fewer complications compared to the multi moment portfolio optimization. Moreover, this paper is an extension of the book chapter of Jurczenko, Maillet and Merlin (2006).

Application research is scarce in this area of research. For example, Taylan and Tathdil (2010) apply the four-moment optimization and efficiency evaluation of Jurczenko, Maillet and Merlin (2008) model on the Turkish stock market. Authors observe stocks which constitute the stock market index ISE-30. The time span in the sample included daily data in 2009, with results indicating major changes in portfolio structure and its moment's characteristics when including higher moments into the analysis.

Besides higher moments in the analysis, a dynamic component in the model has been included as well. However, this was proven to be a difficult task. Briec and Kerstens (2009b) observe performance measurement in a multi-horizon case. This research extends the initial work of Briec, Kerstens and Lesourd (2004) over a multi-horizon time span. However, authors are aware that the limitation of this approach is that portfolio performance measuring is evaluated retrospectively. Some other problems have occurred within this methodology over time as well. Briec and Kerstens (2009a) found infeasibilities when using the approach of Briec and Kerstens (2009b) when evaluating efficiency over time in the context of mean-variance and mean-variance skewness of a portfolio over time. Andriamasy, Briec and Rakotondramaro (2017) introduced the Luenberger-Hicks-Moorsten productivity indicator to measure efficiency over time, due to infeasibility problems of the original Luenberger portfolio productivity indicator. This newer indicator

(L-H-M) allows the researcher to differentiate changes in performance with respect to return and risk strategies. Authors provide an empirical example on ETFs for a two-year period (February 2012 – August 2014) in the study as well. Research is expanding upon the geometrical representation and interpretation of the efficient frontiers which are a result of the optimization of the shortage/distance function. Kerstens, Mounir and Van de Woestyne (2010) provided a geometrical representation of the three moment's portfolio frontier since the risk-return frontier is trivial to interpret. Kerstens, Mounir and Van de Woestyne (2012) analysed consequences of changing the direction vector in ranking the portfolios. They used the following direction vectors: fixed, unit length fixed, position dependent and unit position dependent. This research is (as many within this framework) mostly theoretical, with an empirical example at the end. Authors observed 78 stocks from Euronext from May to October 2009 and simulated 20 portfolios in order to evaluate their performance and provide ranking by using different direction vectors. The results indicate that all direction vectors had their advantages and pitfalls compared to others. Thus, the authors concluded that the ideal scheme does not exist yet. Briec and Kerstens (2010) developed this methodology for the case of partial portfolio moments. This paper includes an empirical application as well (the sample included is 30 blue-chip stocks on LSE, period: January 1990 - May 2001).

Based on this review of previous research, several conclusions can be made. Firstly, the research area of combining efficiency measurement within the portfolio selection methodology is a relatively new area. Thus, there exist many gaps in theory and applications within this field. Future work will surely expand the research on choosing the best direction, if not from an optimization point of view, but maybe from an investment point of view (regarding investor's utility and transaction costs when rebalancing of portfolios is needed). Empirical work could be extended on real investment portfolios in order to get more insights into real investment world problems which could occur when applying this methodology on stock markets. Since the portfolio theory is extending towards partial moments of portfolio distribution in evaluating the performance of financial assets, this methodology could expand on partial moments as well. It seems that much work has to be done both on the theoretical and empirical part of this area of research.

3. Methodology

It is assumed that investor has data on N stocks, where $E(r_i)$ denotes expected return and σ_i^2 denotes the variance of the i -th stock, $i \in \{1, 2, \dots, N\}$. In the Markowitz (1952, 1959) methodology, expected portfolio return is calculated as the weighted return

$$E(R_p) = \sum_{i=1}^N w_i E(r_i) \text{ and the portfolio variance is calculated as the following expression: } \sigma_p^2 = \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij}$$

w_i (and w_j) denotes weight of the i -th (j -th) stock in the portfolio, σ_{ij} is the covariance between return on i -th and j -th stocks. In order to construct the efficient frontier, the following problem is solved:

$$\begin{aligned} \max_{w_i} \quad & E(R_p) = \sum_{i=1}^N w_i E(r_i) \\ \text{s.t.} \quad & \sigma_p = \sqrt{\sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij}} \leq c, \\ & \sum_{i=1}^N w_i = 1 \end{aligned} \tag{1}$$

in which a constraint on every weight can be added, such as, e.g. $w_i \geq 0 \forall i$ (short-selling is excluded) or $a \leq w_i \leq b$, where $a < b$. c is the level of risk investor is willing to accept in order to maximize the portfolio return. Bricc, Kerstens and Lesourd (2004) define the mean-variance representation set,

$$\mathfrak{R} = \{\mathfrak{X} + (\mathbb{R}_+ \times (-\mathbb{R}_+))\} \cap \mathbb{R}_+^2$$

with the Markowitz mean-variance representation of $\mathfrak{X} = \{(\sigma_p, E(R_p)) : w_i \in S\}$, where set S denotes all of the portfolio weights which satisfy the constraints in (1). This set \mathfrak{X} is extended with a cone in order to achieve a convex set for the optimization problem. Proof that set \mathfrak{R} is convex and closed can be seen in Bricc, Kerstens and Lesourd (2004:7-8). The weakly efficient frontier is defined as the following subset:

$$\begin{aligned} \partial^M = \{ & (\sigma_p, E(R_p)) : w_i \in S \wedge (-\sigma_p, E(R_p)) \\ & < (-\sigma'_p, E(R'_p)) \notin \mathfrak{R} \}. \end{aligned} \tag{2}$$

Moreover, if we are evaluating a portfolio k , with its know structure (weights of each stock), risk and return, i.e. $(\sigma_p^k, E(R_p^k))$ is known, the efficiency improvement possibility (EIP) function is defined as:

$$S_g(\mathbf{w}) = \sup \left\{ \delta : (\sigma_p^k - \delta g_\sigma, E(R_p^k) + \delta g_{E(R_p)}) \in \mathfrak{R} \right\}, \tag{3}$$

where \mathbf{w} denotes vector of portfolio weights, δ the rate of risk reduction and return increase needed to arrive on the efficient frontier. This is the shortage/distance function in terms of the Luenberger theory. δ^* is called the portfolio efficiency index (PE) as well, $PE(\mathbf{w}) = \delta^*$. This value determines how much inefficient a portfolio is compared to the efficient frontier. Detailed properties of function given in (3) can be seen in Bricc, Kerstens and Lesourd (2004:10) as well.

Vector $(-g_\sigma, g_{E(R_p)})$ is the direction vector which investor chooses in order to improve efficiency of his portfolio. For a given portfolio $(\sigma_p^k, E(R_p^k))$, a more detailed representation of problem (3) can be written as:

$$\begin{aligned} S_g(\mathbf{w}) = \sup \quad & \delta \\ \text{s.t.} \quad & \sum_{i=1}^N w_i E(r_i) \geq E(R_p^k) + \delta g_{E(R_p)} \\ & \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} \leq \sigma_p^k - \delta g_\sigma \\ & \sum_{i=1}^N w_i = 1 \end{aligned}, \tag{4}$$

with possibility of other constraints on the portfolio weights.

In order to define the overall efficiency index (OE), investor needs to solve the following problem from the utility theory in the first step:

$$\begin{aligned} U^*(\mu, \rho) = \sup \quad & \left[\mu \sum_{i=1}^N w_i E(r_i) - \rho \sum_{i=1}^N \sum_{j=1}^N w_i w_j \sigma_{ij} \right] \\ \text{s.t.} \quad & \sum_{i=1}^N w_i = 1 \end{aligned}, \tag{5}$$

where parameters μ and ρ depend upon investor's risk aversion. In the second step, distance is measured from the initial portfolio $(\sigma_p^k, E(R_p^k))$ and that which is the result from problem (5):

$$\sup \left\{ \delta : (\mu [E(R_p^k) + \delta g_{E(R_p)}] - \rho [\sigma_p^k - \delta g_\sigma]) \leq U^*(\mu, \rho) \right\} \tag{6}$$

where optimal value in (6) is defined as the OE index. The final index which is defined in this approach is the allocative efficiency (AE) index. It is simply the difference between the overall and portfolio efficiency: $AE(\mathbf{w}) = OE(\mathbf{w}) - PE(\mathbf{w})$. The AE index defines how much of the inefficiency of a portfolio is a result of wrong allocation (structure of portfolio). Overall

efficiency is the sum of both sources of inefficiencies. Thus, by solving a couple of quadratic programming problems, the investor can get information on the size of the inefficiency of his portfolio, as well as the sources of that inefficiency. The next section illustrates this on empirical data.

4. EMPIRICAL RESEARCH

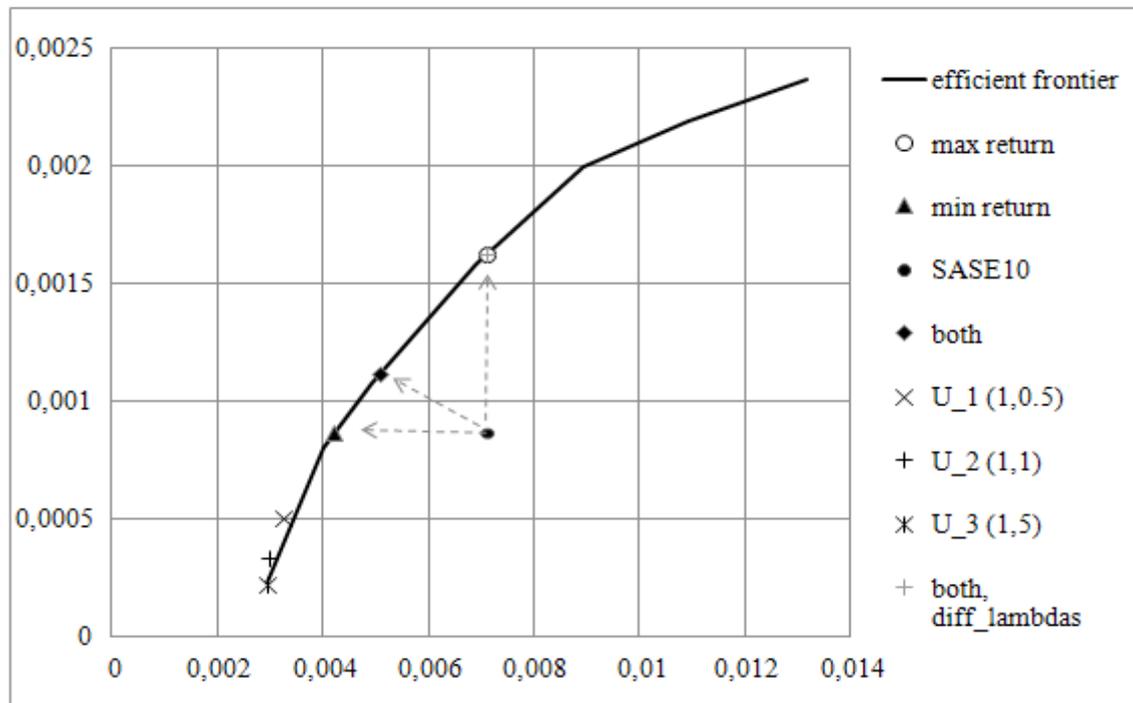
For the purpose of empirical research on real data, daily prices on stocks which constitute the SASE10 index, as well as the daily value of the mentioned index were collected from the web page of Sarajevo Stock Exchange (SSE, 2018). The sample consists of the most recent values from the last restructuring of the market index, ranging from July 2nd 2018 until August 31st 2018. Daily values on index SASE10, as well as prices on stocks which constitute this index in the observed period (BHTRS, BSNLR, ENISR, FDSSR, JPESR, RMUBR, SOLTRK3, SOSOR, UNPLR and ZGPSR). Returns were calculated as continuously compounded returns. Although the observed market is illiquid, existing literature found that illiquidity in terms of Efficient Market Hypothesis has its advantages. Baele, Bekaert and Schäfer (2015) in a detailed review of Central and Eastern European stock markets found that illiquidity premiums exist on those markets. Bloomfield, O'Hara and Saar (2009); Linnainmaa (2007), Tetlock (2008, 2011) found that mispricing is sometimes greater on liquid markets when compared to illiquid ones such as the SSE market. The results are robust when controlling for security characteristics. Moreover, Carhart (1997) has shown that abnormal performance of some institutional funds is consistently better than others, due to having lower transaction costs (by being lower liquid funds). This means that using such methodology as observed here is possible even with problems such as illiquidity.

Before evaluating the efficiency of SASE10 index, an efficient frontier was constructed based upon the formula (1), with the addition of constraints on the stock weights. Since the composition of the market index is defined by the Sarajevo exchange, we limit the maximum weight to be 20% of each stock and limit the minimum weights to the minimal value which was in the observed period to be 0.7385%. In that way, we exclude the possibility of some stocks not entering the portfolio at all, since they all have to be within the market index.

After the construction of the efficient frontier, several EIP functions have been optimized. We started with two basic ones: maximizing the portfolio return

with the risk being equal to the SASE10 index (i.e. a directional function with respect only to the return) and minimizing the portfolio risk with the return being equal to the SASE10 index (i.e. a directional function with respect only to the risk). We call these two portfolios MAX_return and MIN_risk respectively. These two optimization problems are in line with Morey and Morey (1999) measures of maximum expansion of outputs and maximum input reduction. Next, the function in (4) is being evaluated, i.e. the Luenberger shortage function as defined in Briec, Kerstens and Lesourd (2004), called BOTH. Since this specification forces the risk reduction and return increase in the same proportions, we weaken this restriction to have different rates of risk reduction and return increase with model and portfolio called BOTH, diff_lambdas. Another stimulus to observe different rates for input reduction and output expansion can be found in Subhash (2004): in the optimal solution of (4), there could be positive slacks for inputs and/or outputs. This means that the projection of the inefficient portfolio could result in a new portfolio which is not on the efficient frontier. Finally, last three portfolios are constructed by maximizing the investor's utility function starting from SASE10 composition to restructure it to the compositions which give maximum utility when the risk aversion is defined with parameters ($\mu = 1, \rho = 0.5$), ($\mu = 1, \rho = 1$) and ($\mu = 1, \rho = 5$), in order to account for different risk-averse investors. These three portfolios are denoted with U_1, U_2 and U_3 respectively.

The graphical representation of the efficient frontier and each portfolio is shown in Graphic 1. It can be seen that the SASE10 portfolio is not efficient in the observed period. However, efficiency improvements can be made on to any point on the efficient frontier. In this research, we do not focus on the selection of the direction vector for arriving on the frontier. This could be said that depends on the investor's preferences. The focus will be made on the basic performance of each described portfolio and the consequences of choosing to rebalance the SASE10 portfolio. It can be seen that if the investor chooses to maximize the return or to choose the portfolio (BOTH diff_lambdas), he will arrive at the same point. However, the distance between this portfolio and the initial inefficient one is greater compared to the portfolio BOTH. It is an interesting result that if the investor chooses to maximize his utility, all of the three portfolios are on the north-south part of the efficient frontier, where risks are lower compared to the rest of the frontier.

Graphic 1: Efficient frontier and portfolio SASE10 with projections on the frontier

Next, portfolio structure, risk and return are observed in Table 1. Highest return to risk ratio (denoted with R/σ) have portfolios MAX_return and BOTH diff_lambdas. This means that for an additional unit of risk investor has to accept, he can be rewarded with 0.23 units of return. This could be a starting point for the investor to choose the direction in which he wants to arrive on the efficient frontier. If we focus on the portfolio structures, it can be seen that great differences exist between some of the portfolios compared to the original SASE10 one. For example, in the SASE10 index, stock BHTSR and BSNLR have each 20% in the

total composition. But in the portfolio MAX_return, they both have less than 3% and 6% respectively. The opposite is true for stock ZGPSR. This leads to very different positioning in Graphic 1. In order to evaluate sources of inefficiencies of SASE10 portfolio in the observed period, an example of the BOTH portfolio has been chosen as a benchmark. This is due to the minimal distance between these two portfolios. Overall, allocative and portfolio inefficiencies have been calculated by varying the risk aversion as for the three utility based portfolios.

Table 1: Portfolio composition, risk and return for analyzed portfolios

Portfolio:	SASE10	MAX_return	MIN_risk	BOTH	BOTH, diff lambdas	U_1	U_2	U_3
Return	0.0009	0.0016	0.0009	0.0011	0.0016	0.0005	0.0003	0.0002
Standard deviation	0.0071	0.0071	0.0042	0.0051	0.0071	0.0033	0.0030	0.0029
R/σ	0.1224	0.2289	0.2065	0.2201	0.2289	0.1568	0.1134	0.0779
BHTSR	0.2000	0.0253	0.0851	0.0659	0.0352	0.1165	0.1250	0.1308
BSNLR	0.2000	0.0534	0.1328	0.1079	0.0677	0.1667	0.1681	0.1692
ENISR	0.1321	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
FDSSR	0.2000	0.0074	0.0074	0.0074	0.0074	0.0373	0.0512	0.0598
JPESR	0.0663	0.0074	0.0074	0.0074	0.0074	0.0074	0.0074	0.0074
RMUBR	0.0793	0.1693	0.0837	0.1120	0.1732	0.0444	0.0220	0.0074
SOLTRK3	0.0339	0.1289	0.0657	0.0846	0.1002	0.0074	0.0074	0.0074
SOSOR	0.0569	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
UNPLR	0.0241	0.0083	0.0180	0.0149	0.0089	0.0204	0.0190	0.0181
ZGPSR	0.0074	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000

The results are shown in Table 2. The interpretation for the first row is as follows. The overall efficiency of SASE10 portfolio can be improved by 35.40% by increasing return and decreasing risk. Inefficient performance is a result of the inefficient allocative performance of 6.87% (the wrong composition of the portfolio) and 28.52% of portfolio performance (being under the efficient frontier). Thus, if investor chooses to rebalance his portfolio in order to achieve maximal utility with parameters ($\mu = 1$, $\rho = 0.5$), the initial SASE10 portfolio is the least inefficient compared to that new portfolio (with the least improvement needed to achieve this point); and he can achieve maximal utility with the closest portfolio composition. Similar interpretations can be made for the other two rows in Table 2.

Table 2: Sources of inefficiencies of SASE10 portfolio

Utility	Overall efficiency	Allocative efficiency	Portfolio efficiency
$\mu = 1, \rho = 0.5$	0.35396	0.06874	0.2852
$\mu = 1, \rho = 1$	0.64176	0.35654	
$\mu = 1, \rho = 5$	0.92150	0.63628	

Finally, since rebalancing investment portfolios means that investor has to suffer transaction costs, this was included in the analysis as well. The main results are shown in Table 3. Following the methodology of Amenc et al. (2010), we calculate annualized returns, excess returns over the original SASE10 portfolio and

indifference transaction costs for each of the portfolios from the previous calculations. Indifference transaction costs here measure how much the investor can tolerate additional costs of holding the alternative portfolio compared to the initial SASE10 one. If they are positive, this means that the return of the alternative portfolio is of such value that it can cover additional costs of rebalancing the portfolio. The best annualized return was achieved with the portfolio BOTH, diff_lambdas and MAX_return. In total, four out of seven portfolios had equal or greater annualized return compared to the SASE10 portfolio. The excess turnover needed to rebalance each portfolio column shows that portfolio BOTH had the least amount of additional rebalancing needed in order to achieve that portfolio. This is not surprising due to this point being the closest to the original SASE10 one. Since BOTH portfolio had the least amount of needed rebalancing, the indifference transaction costs are the greatest for this portfolio.

As a final summarization of results, it can be stated that if the initial portfolio structure is inefficient, the shortage function derived within the methodology described in this research can evaluate the sources and the size of those inefficiencies. Investor's preferences play one of the main roles in choosing the direction in which improvements should be made. This research shows some possibilities by changing the risk aversion of the investor, as well as using additional transaction costs and annualized return as starting points on how to choose the direction in which the restructuring should go.

Table 3: Annualized returns and indifference transaction costs

Portfolio:	Yearly return (%)	Excess return (%)	Excess turnover (%)	Indifference transaction costs
SASE10	0.0145	-	-	-
MAX_return	0.0271	0.0126	17.75	0.07102
MIN_risk	0.0145	0	12.04	0
BOTH	0.0186	0.0041	2.60	0.1587
Both, diff lambdas	0.0271	0.0126	12.78	0.09859
U_1	0.0085	-0.006	19.27	-0.03104
U_2	0.0057	-0.0088	19.27	-0.04583
U_3	0.0038	-0.0107	19.27	-0.05551

5. CONCLUSION

Successful portfolio management seeks extensive knowledge in the field of quantitative methods and models, as well as finance theory. Investors seek to improve portfolio performance at all times in order to achieve their goals quickly, efficiently and with the smallest costs possible. This paper explored the aspect of measuring inefficiency of a portfolio from a DEA standpoint. This means that for a given portfolio risk, return and its structure, an investor can analyse how much this portfolio is away from the efficient frontier in terms of Modern Portfolio Theory.

Main contributions of the paper are the following ones. Firstly, a brief overview of the relevant literature has been done in one place. In that way, interested readers (investors) could have all relevant research to follow up for more details. Next, an empirical analysis was done within this methodology for the first time on the Sarajevo Stock Exchange, as well as many other surrounding markets. A detailed discussion provided interpretations for (potential) investors in order to repeat this procedure on their own portfolios. However, some of the pitfalls of the research were as follows. A brief time span was included in the analysis. This is due to the latest restructuring of the SASE10 market index when writing this paper. Moreover, a basic model was observed by including only risk and return as main portfolio moments which were observed and assumed that investor cares for. Post Modern Portfolio Theory explores higher moments in the analysis as well. Moreover, due to the unavailability of data, we explored an example of a market index and not concrete portfolios (from individual or institutional investors). Although, usage of this real data is better compared to simulating non-existing portfolio structures. Moreover, the majority of the references used in this study use a market index portfolio as well; meaning that there is a problem with available data in other countries as well.

Future work is going to include evaluation out of sample, as well as we will try to get real investment world data in order to rank real portfolios. Moreover, extended models will be observed, by including higher portfolio moments, as well as dynamic optimization and evaluation of efficiency will be included as well. Since this research is one of the first on these matters on South and East European stock markets, there is hope that it will induce more interest in the future.

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